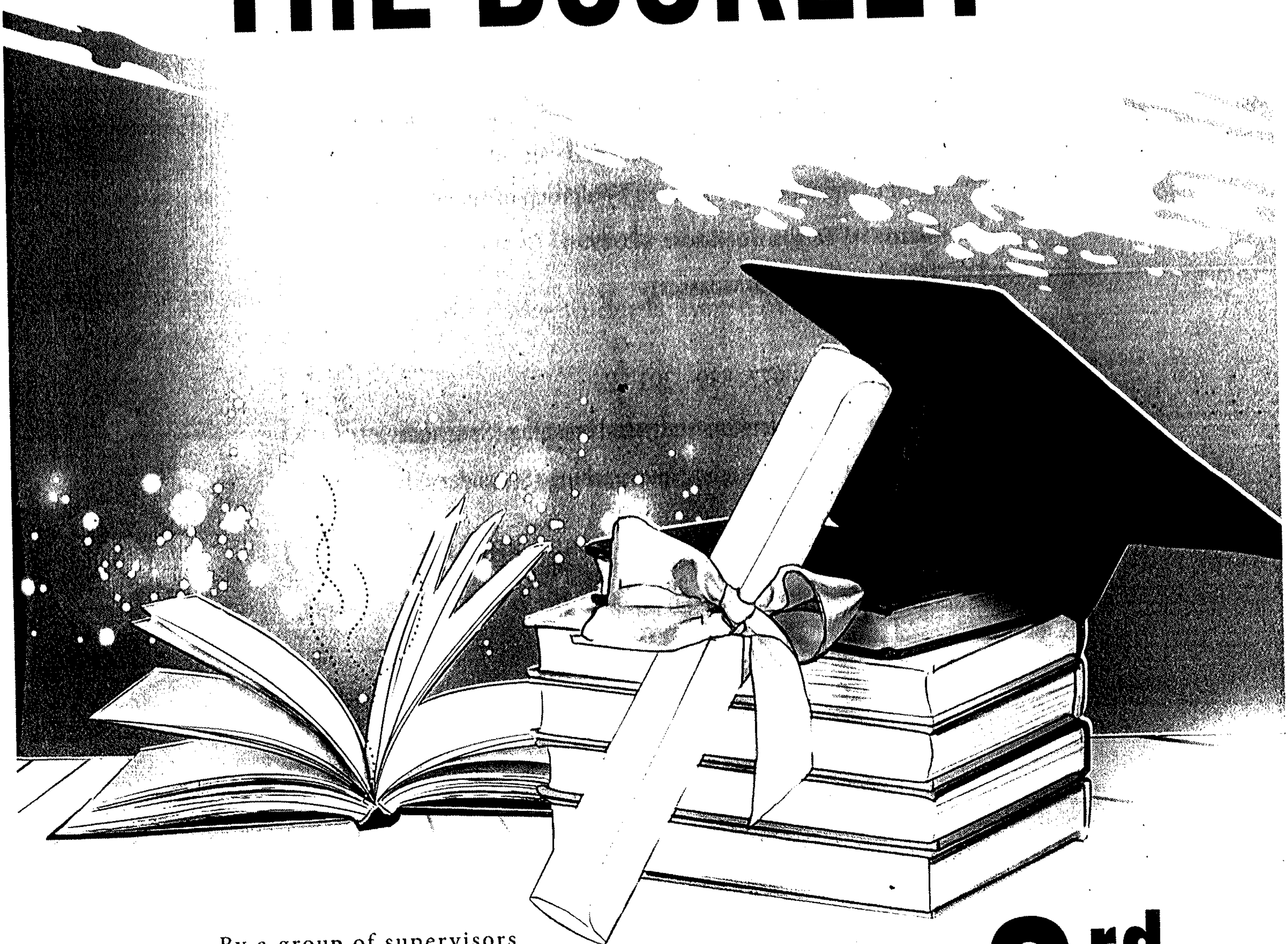
EL-MOASSER

in

Differential & Integral Calculus

Final Examinations

THE BOOKLET



By a group of supervisors

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Preface

Thanks to God who helped us to introduce one of our famous series "El Moasser" in mathematics.

We introduce this book to our colleagues.

We also introduce it to our students to help them study mathematics.

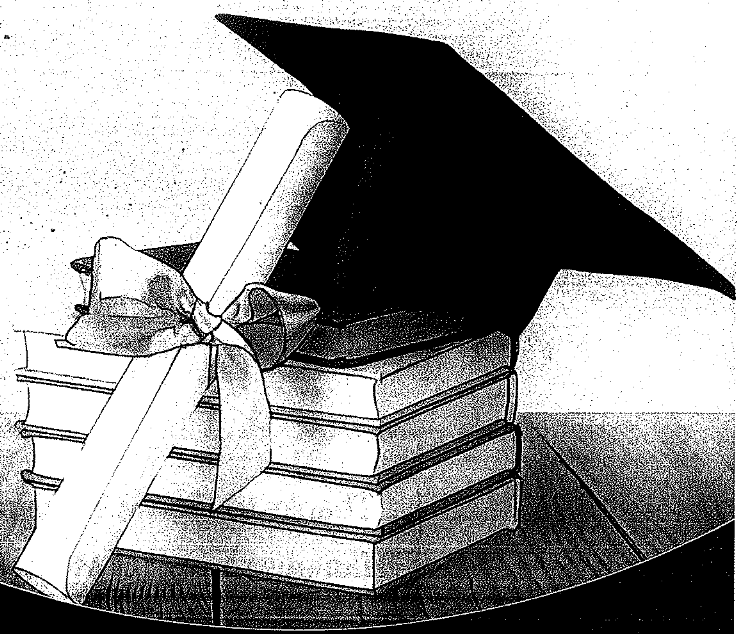
In fact, this book is the outcome of more than thirty years experience in the field of teaching mathematics.

This book will make students aware of all types of questions.

We would like to know your opinions about the book hoping that it will win your admiration.

We will be grateful if you send us your recommendations and your comments.

The Authors



Contents

Egypt Exams and Model Examinations

Each model examination contains 18 questions as the following :

- **10 multiple choices questions.**
- **8 essay questions.**

Remark : Among the 8 essay questions there are two questions each of them contains two requirements, student should answer only one of the two requirements.

تعليمات مهمة

– عدد أسئلة كراسة الامتحان (١٨) سؤالًا.
– عدد صفحات كراسة الامتحان (....) صفحة.
– تأكد من ترقيم الأسئلة ، ومن عدد صفحات كراسة الامتحان ، فهى مسئوليتك.
– زمن الاختبار (ساعتان).
– الدرجة الكلية للاختبار (٣٠) درجة.

عزيزى الطالب ... اقرأ هذه التعليمات بعناية :

١ اقرأ التعليمات جيدًا سواء فى مقدمة كراسة الامتحان أو مقدمة الأسئلة ، وفى ضوئها أجب عن الأسئلة.

٢ اقرأ السؤال بعناية ، وفكر فيه جيدًا قبل البدء فى إجابته.

٣ استخدم القلم الجاف الأزرق للإجابة ، والقلم الرصاص فى الرسومات ، وعدم استخدام مزيل الكتابة.

٤ عند إجابتك للأسئلة المقالية ، أجب فى المساحة المخصصة للإجابة وفى حالة الحاجة لمساحة أخرى يمكن استكمال الإجابة فى صفحات المسودة مع الإشارة إليها ، وإن إجابتك بأكثر من إجابة سوف يتم تقديرها.

مثال :

____

____

____

٥ عند إجابتك عن الأسئلة المقالية الاختيارية أجب عن [a] أو [b] فقط.

٦ عند إجابتك عن أسئلة الاختيار من متعدد إن وجدت :

ظلل الدائرة ذات الرمز الدال على الإجابة الصحيحة تظليلًا كاملًا لكل سؤال.

مثال : الإجابة الصحيحة (c) مثلًا.

ⓐ	
ⓑ	
● c	الإجابة الصحيحة مثلًا
ⓓ	

– فى حالة ما إذا أجبت إجابة خطأ ، ثم قمت بالشطب وأجبت إجابة صحيحة تحسب الإجابة صحيحة.
– وفى حالة ما إذا أجبت إجابة صحيحة ، ثم قمت بالشطب وأجبت إجابة خطأ تحسب الإجابة خطأ.

ملحوظة :

فى حالة الأسئلة الموضوعية (الاختيار من متعدد) إذا تم التظليل على أكثر من رمز أو تم تكرار الإجابة ؛ تعتبر الإجابة خطأ.

٧ يسمح باستخدام الآلة الحاسبة.

Egypt exams

Egypt Exam 1st session 2017 on differential and integral calculus

Answer the following questions :

ax

1 If the function $f : f(X) = X + \frac{a}{X}$ has a critical point at $X = 2$, then the value of a = ...

$f'(x) = 1 - x^{-2} a$

$1 - (2)^{-2}(a) = 0$

$1 = \frac{a}{4}$

$a = 4$

(a) 4
(b) 3
(c) 2
(d) 1

2 If the curve of the function $f : f(X) = \cos X - a X^2$ has an inflection point at $X = \frac{\pi}{3}$, then the value of a = ...

$f'(x) = -\sin x - 2ax$

$f''(x) = -\cos x - 2a$

$-\cos 60 = 2a$

$-\frac{1}{2} = 2a$

$4a = -1$

$a = -\frac{1}{4}$

(a) $\frac{1}{4}$
(b) $-\frac{1}{4}$
(c) $\frac{1}{2}$
(d) -1

3 The absolute maximum value of the function f such that : $f(X) = \sin X + \cos X$ in the interval $[0 , 2\pi]$ is ....

$f'(x) = -\cos x + \sin x$ put $f'(x) = 0$

$\cos x = \sin x$ $\therefore x = 45°$

$f(x) = \sin(45) + \cos 45 = \sqrt{2}$

$f(x) = \sin(0) + \cos(0) = 1$

$f(x) = \sin 360 + \cos(360) = 1$

(a) zero
(b) $\frac{1}{\sqrt{2}}$
(c) 1
(d) $\sqrt{2}$

4 ***Answer one of the following items :***

[a] Determine the local maximum values and the local minimum values (if there exist) for the function $f : f(X) = (2 - X)\, e^{X}$

[b] Find the absolute maximum value and the absolute minimum value of the function f such that : $f(X) = 3X^4 - 4X^3$ in the interval $[-1, 2]$

a) $F(x) = (2-x)e^x$, $\therefore F'(x) = (2-x)e^x + e^x(-1)$
$= (2-x)e^x - e^x$
$= e^x(2-x-1)$
Put $F'(x) = 0$ $= e^x(1-x)$
$e^x = 0$ ref $\quad 1-x=0 \quad x=1$
$+ + + + \mid - - - - \; F'(x)$
it's increasing $]-\infty, 1[$ & decreasing $]1, \infty[$

b)
$F(x) = 3x^4 - 4x^3 \qquad [-1, 2]$
$F'(x) = 12x^3 - 12x^2$
Put $F'(x) = 0 \quad 12x^3 = 12x^2 \quad \therefore x = 1$ or zero
at $x = 1 \quad \therefore f(x) = 3(1)^4 - 4(1)^3 = -1 \longrightarrow$ abs min.
at $x = 0 \quad \therefore f(x) =$ zero
at $x = 2 \quad \therefore f(x) = 16 \longrightarrow$ absolut max.
at $x = -1 \quad \therefore f(x) = 7$

5 $\int 2\cos^2 X \, dX =$ ....

$\cos 2x = \cos^2 x - \sin^2 x$
$= 2\cos^2 x - 1$
$= 1 - 2\sin^2 x$

(a) $X + \frac{1}{2}\sin 2X + c$

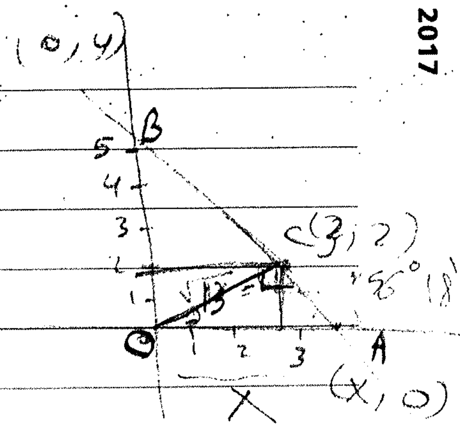
(b) $X + 2\sin 2X + c$

(c) $X - \frac{1}{2}\sin 2X + c$

(d) $X - \sin 2X + c$

6 In the orthogonal coordinate plane , the straight line $\overleftrightarrow{AB}$ is drawn passing through the point C (3 , 2) , cutting the positive part of X-axis at the point A and the positive part of y-axis at the point B , find the smallest area for Δ AOB such that O is the origin point.

Point A = (x, 0)
point B = (0, y)
Point O = (0, 0)



$\frac{2-y}{3-x} =$

$\frac{2-0}{3-x} = \frac{y-2}{0-3}$

$\sin\theta = \frac{2}{\sqrt{13}}$

$\frac{2}{3-x} = \frac{y-2}{3}$

$\alpha = 180 - (90 + 56°$
$= 33°41'$

$6 = (y-2)(3-x)$

7 If $f(X) = |X|$, then $\int_{-2}^{2} f(X)\, dX = \dots\dots\dots\dots$

calculator

(a) 4

(b) 2

(c) 0

(d) − 1

$\begin{cases} x & x \geq 2 \\ x & x < -2 \end{cases}$

$\frac{1}{3}x^3 \Big]_{-2}^{2} = \frac{1}{3}(2)^3 - \frac{1}{3}(-2)^3$

8 Find the area of the region bounded by the two curves :
$y = x^2$, $y = 5x$

By solving Two equations:-

$x^2 = 5x$, $x^2 - 5x = 0$

$x = 5$ $x = 0$

$y = (5)^2 = 25$

$y = 5(5) = 25$

$\left| \int_0^5 x^2 - 5x \, dx \right| = \left[\frac{1}{3}x^3 - \frac{5}{2}x^2 \right]_0^5$

$= \left| \frac{1}{3}(5)^3 - \frac{5}{2}(5)^2 \right| + \left| \frac{1}{3}(0)^3 - \frac{5}{2}(0)^2 \right|$

$= \frac{125}{6}$ square unit

9 Find the volume of the solid generated by revolving the region bounded by the two curves : $y = x^2$, $y = 3x$ a complete revolution about the x-axis

By solving the equations: $x^2 = 3x$

$x^2 - 3x = 0$

$x = 3$ $x = 0$

$V = \pi \int_0^3 (x^2)^2 - (3x)^2 \, dx$

$= \pi \int_0^3 x^4 - 9x^2 \, dx$

$= \pi \left[\frac{1}{5}x^5 - 3x^3 \right]_0^3 = \pi \left(\left(\frac{1}{5}(3)^5 - 3(3)^3 \right) - \left(\frac{1}{5}(0)^5 - 3(0)^3 \right) \right)$

$= \frac{162}{5} \pi$ cubic units

10 ***Answer one of the following items :***

[a] Find : $\int \frac{x}{x+1}\, dx$

[b] Find : $\int x^2 \ln x \, dx$

(a) let $z = x+1$ $\therefore dz = dx$

$x = z-1$, $\int \frac{z-1}{z} dz = \int 1 - \frac{1}{z} dz$

$= z - 1 + C = x + 1 - 1 + C = x + C$

(b) $\int x^2 \ln x \, dx$ x^2 $\ln x$

$\frac{x^3}{3} \ln x - \int \frac{1}{3} x^2 \, dx$ $\frac{1}{3}x^3$ $\int \frac{1}{x}$

$= \frac{x^3}{3} \ln x - \frac{1}{9} x^3 + C$

11 If $f(x) = a\,e^{x}$, then $f'(-2) = \ldots\ldots\ldots\ldots$

$F'(x) = a\,e^{x}$

(a) $-f(2)$

(b) $-f'(2)$

(c) $-f(-2)$

(d) $f(-2)$

12 $\int \frac{\ln x^2}{\ln x}\, dx = \ldots\ldots\ldots\ldots$

$\int \frac{2 \ln x}{\ln x} dx = \int 2\, dx$

$= 2x + C$

(a) $\frac{x}{2} + c$

(b) $\frac{1}{x} + c$

(c) $2x + c$

(d) $\ln |x| + c$

13 $\int \cot x \, d x =$ ....

(a) $\ln|\sin x| + c$

(b) $\ln|\cos x| + c$

(c) $-\ln|\sin x| + c$

(d) $\ln|\csc x| + c$

$\int \cot x \, dx$

$= \int \frac{\cos x}{\sin x} \, dx$

$= \ln|\sin x| + c$

14 Find the equation of the normal to the curve $y = 3e^{x}$ at the point lying on it and its x-coordinate equals -1

$y = 3e^{-1}$

$\frac{dy}{dx} = 3e^{x}$, slope $= 3e^{-1}$

eq. of normal: $\frac{y - y_p}{x - x_p} = \frac{-1}{3e^{-1}} = \frac{-e}{3}$

$\frac{y - \left(\frac{3}{e}\right)}{x + 1} = \frac{-e}{3}$

$3y - \frac{1}{e} = -ex - e$

$-ex + 3y = \frac{1}{e} - e$

15 If $y = \cot\left(\frac{\pi}{6} t\right)$, $t = 3\sqrt{x}$, then $\left(\frac{dy}{dx}\right)_{x=1} =$ ....

(a) $\frac{-\pi}{4}$

(b) $\frac{-\pi}{9}$

(c) $\frac{-\pi}{6}$

(d) $\frac{\pi}{4}$

$y = \cot\left(\frac{\pi}{6}(3\sqrt{x})\right)$

$= \cot\left(\frac{\pi}{2} \cdot \sqrt{x}\right)$

$\frac{dy}{dx} = -\csc\left(\frac{\pi}{2}\sqrt{x}\right) \times \frac{1}{2\sqrt{x}} \cdot \frac{\pi}{2}$

16 The slope of the tangent to the curve $x y^2 = 3$ at the point $(3 , 1)$ equals ....

(a) -6

(b) -3

(c) $\frac{-1}{6}$

(d) $\frac{1}{3}$

$x\left(2y\frac{dy}{dx}\right) + y^2 = 0$

$2xy\frac{dy}{dx} = -y^2$

$\frac{dy}{dx} = \frac{-y^2}{2xy} = \frac{-y}{2x} = \frac{-(1)}{2(3)} =$

17 If $x = \frac{z+1}{z-1}$, $y = \frac{z-1}{z+1}$, **find :** $\frac{d^2 y}{d x^2}$ at $z = 0$

$y = \frac{1}{x} = x^{-1}$

$\frac{dy}{dx} = \frac{-1}{x^2} = -x^{-2}$

$\frac{d^2y}{dx^2} = \frac{+2}{x^3} = \frac{2}{\left(\frac{0+1}{0-1}\right)^3} = -2$

18 If a stone fell in a settle water lake , then a circular wave is formed whose radius increases at a rate of 4 cm./sec. Find the rate of increasing of the surface area of the wave at the end of 5 seconds.

$\frac{dr}{dt}$

$\text{Area} = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$= 2\pi(20) 4$

$\frac{dA}{dt} = 160\pi \text{ cm}^2/\text{sec}$

after 5 sec

$\therefore r = 4 \times 5 = 20$

Egypt Exam 2^{nd} session 2017 on differential and integral calculus

Answer the following questions :

1 $\int \sec^4 x \tan x \, dx =$ ....

(a) $\frac{1}{5} \sec^5 x + c$

(b) $\frac{1}{4} \sec^4 x + c$

(c) $\frac{1}{3} \tan x + c$

(d) $\frac{-1}{3} \tan^3 x + c$

2 Find the maximum area for the isosceles triangle that could be drawn inscribed in a circle whose radius equals 12 cm.

3 If $f(x) = \sin^3 x$, then $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} f(x)\, dx =$ ....

(a) 4

(b) 2

(c) zero

(d) -1

4 **Find the area of the region bounded by the two curves :**

$y = x^2$, $y = 4x$

5 Find the volume of the solid generated by revolving the region bounded by the two curves :

$y = x^2$, $y = 2x$ a complete revolution about x-axis.

6 ***Answer one of the following items :***

[a] Find : $\int \frac{x}{3x^2 + 1} \, dx$

[b] Find : $\int \frac{x}{e^{2x}} \, dx$

7 If $y = \sec x$, then $\dddot{y}\left(\frac{\pi}{3}\right) = \dots\dots\dots$

(a) $2\sqrt{3}$

(b) 6

(c) 8

(d) 14

8 If $x = 2t^2 + 3$, $y = \sqrt{t^3}$

then $\left(\frac{dy}{dx}\right)_{t=1} = \dots\dots\dots$

(a) $\frac{3}{8}$

(b) 5

(c) $\frac{8}{3}$

(d) 6

9 If $y = x \sin x$

, prove that : $x \frac{d^3 y}{d x^3} + x \frac{dy}{dx} + 2y = 0$

10 A rectangle of length 24 cm. and width 10 cm. , if its length shrinks at a rate of 2 cm./sec. while its width increases at a rate of 1.5 cm./sec. Find the rate of change of its area at the end of 4 seconds , after how many seconds does the area stop increasing ?

11 $\lim\limits_{x \to 0} \frac{2^x - 1}{3x} = \dots\dots\dots\dots$

(a) $3 \ln 2$

(b) $\frac{1}{3} \ln 2$

(c) $\ln \frac{2}{3}$

(d) $2 \ln 3$

12 $\int 4x e^{x^2+1} \, dx = \dots\dots\dots\dots$

(a) $e^{x^2+1} + c$

(b) $4 e^{x^2+1} + c$

(c) $\frac{1}{2} e^{x^2+1} + c$

(d) $2 e^{x^2+1} + c$

13 $\int \frac{\ln x^2}{x \ln x^3}\, dx = \ldots\ldots\ldots\ldots$

(a) $x \ln \frac{1}{x} + c$

(b) $\frac{2}{3 \ln x} + c$

(c) $\frac{2}{3} \ln |x| + c$

(d) $\frac{2}{3x \ln x} + c$

14 If $y = (x^3 + 5)^x$, **find :** $\frac{dy}{dx}$

15 If $f :]-1, 4[\longrightarrow \mathbb{R}$, $f(x) = x^3 - 3x$, then the number of the critical points for the function f equals ....

(a) zero

(b) 1

(c) 2

(d) 3

16 If the curve $y = x^3 + a x^2 + b x$ has an inflection point at $(3 , -9)$,
then $a + b =$ ....

(a) 15

(b) 6

(c) -9

(d) -12

17 The maximum value for the expression : $4 x - x^2$, where $x \in \mathbb{R}$ is ....

(a) 4

(b) 2

(c) 3

(d) 6

18 ***Answer one of the following items :***

[a] Determine the maximum and the minimum local values for the function f such that : $f(x) = x^3 - 3 x^2 - 9 x$, then determine the inflection point (if exists) for the function.

[b] Find the absolute extrema values of the function f such that :
$f(x) = 10 x e^{-x}$, $x \in [0 , 4]$

Egypt Exam 1st session 2018 on differential and integral calculus

Answer the following questions :

1 If $a^y = b^x$ such that $a , b \in \mathbb{R}^+$, $a \neq b$, then $\frac{dy}{dx} = \ldots\ldots\ldots\ldots$

(a) $\log \frac{a}{b}$

(b) $\log_a b$

(c) $\log_b a$

(d) $\log \frac{b}{a}$

2 If $\int_{-2}^{5} f(x)\, dx = 12$, $\int_{-2}^{5} f(x)\, dx = 16$, then $\int_{3}^{5} f(x)\, dx = \ldots\ldots\ldots\ldots$

(a) -28

(b) -4

(c) 4

(d) 28

3 ***Answer one of the following items :***

[a] Find : $\int x^3 (x^2 + 1)^6 \, dx$

[b] Find : $\int (x - 3) e^{2x} \, dx$

4 $\int \tan \theta \, d\theta =$ ....

(a) $-\ln |\cos \theta| + c$

(b) $-\ln \cos \theta + c$

(c) $\ln \cos \theta + c$

(d) $|\ln \cos \theta| + c$

5 $\int_{-\pi}^{\pi} \frac{2x - \sin x}{x^2 + \cos x} \, dx =$ ....

(a) $-\pi$

(b) zero

(c) π

(d) 2π

6 ***Answer one of the following items :***

[a] Find the local maximum values and the local minimum values of the function $f : f(x) = x^3 - 3x - 2$, and the inflection points of the curve of the function (if exists)

[b] Find the absolute extrema values of the function $f : f(x) = x(x^2 - 12)$ in the interval $[-1, 4]$

7 If $f'(x) = x\,f(x)$ and $f(3) = -5$, then $f''(3) =$ ....

(a) -50

(b) 4

(c) 15

(d) 27

8 The curve of the function $f : f(x) = (x - 2)\,e^{x}$, is convex upwards in the interval ....

(a) $]-1, 2[$

(b) $]-\infty, 0[$

(c) $]0, \infty[$

(d) $]0, 2[$

9 **Find the equations of the tangent and the normal to the curve :**

$x = \sec\theta$, $y = \tan\theta$ at $\theta = \frac{\pi}{6}$

10 If $\sin y + \cos 2X = 0$, **prove that :** $\frac{d^2 y}{d X^2} - \left(\frac{d y}{d X}\right)^2 \tan y = 4 \cos 2X \sec y$

11 If $X = 2t^3 - 15t^2 + 36t + 1$, $y = t^2 - 8t + 11$

, then this curve has a vertical tangent at t = ...

(a) 4

(b) 3 or 2

(c) 6

(d) 8

12 For the function f such that $f'(X) = -2X + 6$, then all of the following statements are correct except ....

(a) the curve of the function f convex upwards in the interval $]-\infty , \infty[$

(b) the function f has a local minimum value at $X = 3$

(c) the curve of the function f has no inflection points.

(d) $f(X)$ is decreasing in the interval $]3 , \infty[$

13 If $y = a\,x^b$ such that a and b are constants , **prove that :** $\frac{1}{y} \times \frac{dy}{dt} = \frac{b}{x} \times \frac{dx}{dt}$

14 Find the volume of the solid generated by revolving the region bounded by the curve $y = x^2 + 2$, the x-axis and the two straight lines $x = -2$, $x = 2$ a complete revolution about the x-axis.

15 $\lim\limits_{x \to 0} \left(\frac{2^{x}-1}{3x}\right) = \text{..............}$

ⓐ $3 \ln 2$

ⓑ $\frac{1}{3} \ln 2$

ⓒ $\ln \frac{2}{3}$

ⓓ $2 \ln 3$

16 If $f(X) = X(a - \ln X)$ such that a is constant , the curve of the function has a critical point at $X = e$, then a = ...

ⓐ 1

ⓑ 0

ⓒ e

ⓓ 2

17 A metalic circular sector whose area is 4 cm^2. Find the radius length of the sector's circle which makes its perimeter as minimum as possible.

What is the measure of its angle then ?

18 Find the area of the region bounded by the curve $y = 4 - x^2$

and the straight line $y = x + 2$

Egypt Exam 2nd session 2018 on differential and integral calculus

Answer the following questions :

1 If $f(X) = \sqrt{\sin 2X} - \csc X$, then $f'\left(\frac{\pi}{4}\right) = \dots\dots\dots\dots$

(a) $\sqrt{2}$

(b) 1

(c) zero

(d) -1

2 If the curve : $y = (2X - a)^3 + 4$ has an inflection point at $X = 5$, then $a = \dots\dots\dots\dots$

(a) 2

(b) 4

(c) 5

(d) 10

3 A lake infected by bacteria has been treated by an antibacterial. If the number of bacteria z in 1 cm^3 after n day is given by the relation $z(n) = 20\left(\frac{n}{12} - \ln\left(\frac{n}{12}\right)\right) + 30$ such that $1 \leq n \leq 15$

(1) When the number of bacteria be minimum during this interval ?

(2) What is the least number of bacteria during this interval ?

4 Find the volume of the solid generated by revolving the region bounded by the two curves $y = x^2$ and $y = 3x - 2$ a complete revolution about the x-axis.

5 If $y = e^{(1 + \ln x)}$; then $\frac{dy}{dx} = \dots\dots\dots\dots$

(a) x

(b) $e\,x$

(c) e

(d) 1

6 $\int_{-1}^{1} \frac{x^3}{x^4 + \cos x}\, dx = \dots\dots\dots\dots$

(a) -1

(b) zero

(c) 1

(d) 4

7 *Answer one of the following items :*

[a] Find : $\int x(x+2)^6 \, dx$

[b] Find : $\int (x+5)\, e^x \, dx$

8 $\int \frac{x+2}{x+1} \, dx =$ ....

(a) $1 + \ln (x+1) + c$

(b) $x - \ln |x+1| + c$

(c) $x + \ln (x+1) + c$

(d) $x + \ln |x+1| + c$

9 $\int_{0}^{\frac{\pi}{4}} \sec^2 x \tan x \, dx = \ldots\ldots\ldots\ldots$

(a) zero

(b) $\frac{1}{2}$

(c) 1

(d) 2

10 ***Answer one of the following items :***

[a] Find the local maximum and minimum values (if found) of the function f :

$f(x) = x^4 - 2x^2$

[b] Find the absolute extrema values of the function $f : f(x) = \frac{4x}{x^2 + 1}$

in the interval $[-1, 3]$

11 $\lim\limits_{x \to \infty} \left(1 + \frac{1}{x}\right)^{3x} = \dots\dots\dots\dots$

(a) 1

(b) 3

(c) e

(d) e^3

12 If the curve of the function $f(x) = a\,x^2 + 12\,x + 1$ has a critical point at $x = 2$, then a = ...

(a) 12

(b) -3

(c) -1

(d) 3

13 Find the equations of the tangent and the normal to the curve : $y = 3 + \sec x$ at the point which lies on the curve and its x-coordinate equals $\frac{2\pi}{3}$

14 Find the area of the region bounded by the curve $y = \sqrt{2x}$ and the straight line $y = x$

15 If $y = 2t^3 + 7$, $z = t^2 - 4$, then the rate of change for y with respect to z equals ....

(a) $2t$

(b) $3t$

(c) 6

(d) 12

16 The curve of the function $f : f(x) = (x - 2)e^x$ is convex downwards in the interval ....

(a) $]-\infty , \infty[$

(b) $]-1 , 2[$

(c) $]0 , 2[$

(d) $]0 , \infty[$

17 If $\sin x = x y$, **prove that :** $x^2 (y + y'') + 2 \cos x = 2 y$

18 If $x e^y = 2 - \ln 2 + \ln x$ and $\frac{dx}{dt} = 6$ at $x = 2$, $y = 0$, find $\frac{dy}{dt}$

Model examinations in the booklet form

Model 1

Answer the following questions :

1. $\lim\limits_{x \to \infty} \left(1 + \frac{1}{x}\right)^{2x} = \dots\dots\dots$

   (a) 1
   (b) 2
   (c) e
   (d) e^2

2. The curve $y = x e^{x}$ at ....

   (a) $x = -1$ has minimum value
   (b) $x = -1$ has maximum value
   (c) $x = 0$ has minimum value
   (d) $x = 0$ has maximum value

3. The tangent to the curve $y = 3x^2 - 5$ at the point $(1, -2)$ also passes by the point ....

   (a) $(5, -2)$
   (b) $(3, 1)$
   (c) $(2, -4)$
   (d) $(0, -8)$

4. If the perimeter of a circular sector is P , then its surface area is maximum at r = ...

   (a) $\frac{P}{2}$
   (b) $\frac{1}{\sqrt{P}}$
   (c) $\sqrt{P}$
   (d) $\frac{P}{4}$

5 If $x = e^{2t}$, $y = t^3$ **Find :** $\frac{d^2 y}{d x^2}$

6 If f is a continuous even function on the interval $[-4, 4]$,

$\int_{-4}^{4} f(x)\, dx = 20$, $\int_{0}^{2} f(x)\, dx = 6$

, then $\int_{-4}^{2} f(x)\, dx =$ ....

(a) 120

(b) 14

(c) 26

(d) 16

7 If $f(x) = \cot x$, then $f'''\left(\frac{\pi}{4}\right) =$ ....

(a) $\frac{-4}{9}$

(b) $\frac{4}{9}$

(c) 4

(d) $\frac{9}{2}$

8 ***Answer one of the following items :***

[a] Find : $\int x^2 \ln x \, dx$

[b] Find : $\int (x^{2e} + e^{3x}) \, dx$

9 If $f(x) = 2x^3 - 3x^2 - 12x + 12$

Answer one of the following items :

[a] Find the increasing and decreasing intervals.

[b] Find the local maximum and the local minimum values of the function.

10 Find the equation of the curve passes through the point $(0 , 1)$ and the slope of its tangent at any point on it (x , y) equals $x\sqrt{x^2 + 1}$

11 The length of each side of an equilateral triangle = a , and increase at a rate k , then the rate of increasing of its surface area equals ....

(a) $\frac{2}{\sqrt{3}}$ a k

(b) $\sqrt{3}$ a k

(c) $\frac{\sqrt{3}}{2}$ a k

(d) $\frac{2}{\sqrt{3}}$ a k

12 The curve of the function f is convex downwards in $\mathbb{R}$ if $f(X) =$ ....

(a) $3 - X^2$

(b) $3 - X^3$

(c) $3 - X^4$

(d) $3 + X^4$

13 $\int \tan\theta \, d\theta =$ ....

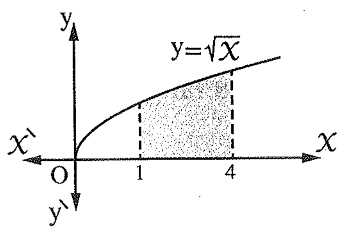
(a) $-\ln|\cos\theta| + c$

(b) $-\ln\cos\theta + c$

(c) $\ln\cos\theta + c$

(d) $|\ln\cos\theta| + c$

14 The volume of the solid generated by revolving the shaded region a complete revolution about X-axis equals .... cube units.



(a) $\frac{14}{3}\pi$

(b) $\frac{15}{2}\pi$

(c) $\frac{15}{2}$

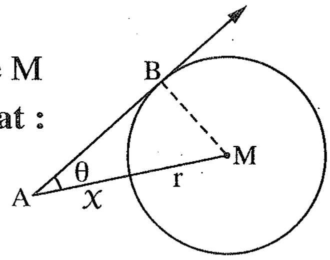
(d) $\frac{14}{3}$

15 Find the shortest distance between the straight line : $x - 2y + 10 = 0$ and the curve $y^2 = 4x$

16 Find the area of the region bounded by the curve of the function $y = x^3$ and the two straight lines $y = 0$, $x = 2$

17 If $x^3 + y^3 = 1$, **prove that :** $y^2 y'' + 2 y y'^2 + 2x = 0$

18 **In the opposite figure :**



A is a point moving on the plane , $\overrightarrow{AB}$ is a tangent of the circle M at B , $AM = x + r$ where r is the radius of the circle , **prove that :** $x = r(\csc\theta - 1)$, then find the rate of change of x with respect to θ at $\theta = \frac{\pi}{6}$

Model 2

Answer the following questions :

1. The function $f : f(x) = \frac{x}{\ln x}$ is increasing in the interval ....
   - (a) $]0 , \infty[$
   - (b) $]0 , e[$
   - (c) $]e , \infty[$
   - (d) $]-\infty , \infty[$

2. The normal to the circle $x^2 + y^2 = 12$

   at any point on it passes through the point ....
   - (a) $(2 , 2)$
   - (b) $(1 , 1)$
   - (c) $(0 , 0)$
   - (d) $(-2 , -2)$

3. $\int (4 - \csc x \cot x) \, dx =$ ....
   - (a) $4x - \csc x + c$
   - (b) $4x + \csc x + c$
   - (c) $4x - \cot x + c$
   - (d) $4x + \cot x + c$

4. $\lim\limits_{x \to \infty} \left(\frac{x+6}{x+2}\right)^{x+5} =$ ....
   - (a) e^4
   - (b) e^5
   - (c) e
   - (d) e^6

5. Find the area of the greatest rectangle which can be drawn in a circle of radius 4 cm.

6. Identify the points of local maximum and local minimum and inflection points if exist to the curve of the function f where : $f(X) = 2X^3 + 3X^2 - 12X + 5$

7 If $\int_{-2}^{2} f(X)\, dX = 0$, then $f(X) =$ ....

(a) $X^2 + 1$

(b) X

(c) $X + 1$

(d) $X - 1$

8 If $f(X) = \sin 2X \cos 2X$, then $f'''\left(\frac{\pi}{3}\right) =$ ....

(a) -4

(b) 0

(c) $4\sqrt{3}$

(d) 8

9 ***Answer one of the following items :***

[a] If $y = (\sin X)^{\tan X}$, find $\frac{dy}{dX}$

[b] If $y = (5 - 3 \csc X)^4$, find $\frac{dy}{dX}$

10 The side length of a cube is equal to the diameter length of a sphere and the rate of increasing of the side length of the cube is equal to the rate of increasing of the radius , then the ratio between the increasing of their areas equals ....

(a) $\frac{\pi}{6}$

(b) $\frac{2\pi}{3}$

(c) $\frac{3}{2\pi}$

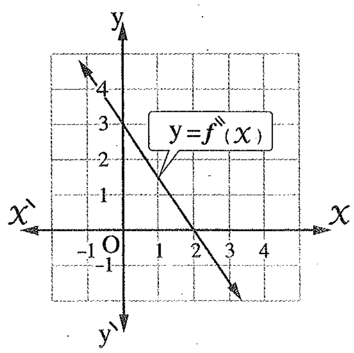
(d) $\frac{3}{\pi}$

11 ***Answer one of the following items :***

[a] Find : $\int x^2 e^x \, dx$

[b] Find : $\int x (x + 2)^8 \, dx$

12 The opposite figure represents the curve of the function f'' , then the curve of the function f has an inflection point at $x =$ ....



(a) 0

(b) 1

(c) 2

(d) 3

13 If $y^2 + a x^3 - b x = c$ where a , b , c are constants

, prove that : $y \frac{d^2 y}{d x^2} + \left(\frac{d y}{d x}\right)^2 + 3 a x = 0$

14 The area of the region bounded by the curve $y = x^3$ and the straight lines $x = -1$, $x = 1$, $y = 0$ equals ....

(a) zero

(b) $\frac{1}{2}$

(c) $\frac{1}{4}$

(d) 6

15 **Find the equation of the curve :**

$y = f(x)$ if $y'' = 6x - 4$ and the curve has local minimum at (1 , 5)

16 The curve of the function $f : f(X) = (X - 2)\, e^{X}$ is convex downwards in the interval ....

(a) $]-\infty , \infty[$

(b) $]-1 , 2[$

(c) $]0 , 2[$

(d) $]0 , \infty[$

17 The rate of increasing of the length of each of two sides in a triangle is 0.1 cm./sec. and the rate of increasing of angle including between them is $\frac{1}{5}$ rad/sec. Find the rate of increasing of area of the triangle at the instant when the length of each side of the triangle is 10 cm.

18 Find the volume of the solid generated by revolving the region bounded by the two curves $y = X^2$ and the straight line passes through the two points $(0 , 6) , (1 , 7)$ a complete revolution about X-axis.

Model 3

Answer the following questions :

1 $\lim\limits_{x \to 0} \dfrac{e^{x} - \sin x - 1}{x} = \ldots\ldots\ldots\ldots$

(a) zero

(b) 1

(c) undefined

(d) -1

2 If the curve of the function f represents a polynomial function , has a local maximum at the point (a , b) , then $f'(a) = \ldots\ldots\ldots\ldots$

(a) b

(b) zero

(c) $\frac{-b}{a}$

(d) undefined

3 If the tangent to the curve $y^2 = 4 a x$ is perpendicular to x-axis , then ....

(a) $\frac{dy}{dx} = 0$

(b) $\frac{dy}{dx} = 1$

(c) $\frac{dx}{dy} = 1$

(d) $\frac{dx}{dy} = 0$

4 If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$, then $\frac{dy}{dx} = \ldots\ldots\ldots\ldots$

(a) $\sin\theta$

(b) $\sin 2\theta$

(c) $\cos\theta$

(d) $\tan\theta$

5 The volume of the solid generated by revolving the region bounded by the two curves $y = \tan X$, $y = \sec X$ and the two straight lines $X = \frac{\pi}{6}$, $X = \frac{\pi}{3}$ a complete revolution about X-axis is .... cubic unit.

(a) $\frac{\pi^2}{6}$

(b) $\frac{\pi^2}{3}$

(c) $\frac{2\pi^2}{5}$

(d) $2\pi^2$

6 If $f : \left[\frac{1}{e}, e\right] \longrightarrow \mathbb{R}$ and $f(X) = X - \ln X$

Answer one of the following items :

[a] Determine the increasing and decreasing intervals.

[b] Find the absolute maximum and absolute minimum of the function.

7 The rate of change for $\sqrt{X^2 + 16}$ with respect to $\frac{X}{X-1}$ at $X = 3$ equals ....

(a) -60

(b) $\frac{-5}{12}$

(c) $\frac{-12}{5}$

(d) $\frac{-3}{5}$

8 $\int \frac{\ln x^2}{\ln x} \, d x =$ .... (where : $x > 0$)

(a) $\frac{x}{2} + c$

(b) $\frac{1}{x} + c$

(c) $2x + c$

(d) $\ln |x| + c$

9 If $x^2 y = a\, b \ln x$ where a , b are constants , **prove that :** $x^2 \frac{d^2 y}{d x^2} + 5 x \frac{d y}{d x} + 4 y = 0$

10 If the rate of change in volume of a sphere equals the rate of change of its radius , then r = ... length unit.

(a) 1

(b) $\sqrt{2\pi}$

(c) $\frac{1}{\sqrt{2\pi}}$

(d) $\frac{1}{2\sqrt{\pi}}$

11 **Find :** $\int x \cos x \, d x$

12 The curve $y = (2x - c)^3 + 4$ has an inflection point at $x = 5$

, then c = ...

(a) 2

(b) 4

(c) 5

(d) 10

13 **Find :** $\int \sqrt{x}\left(1 + \sqrt{x}\right) d x$

14 If $y = e^{x}$, $z = \sin x$

, then $\frac{dy}{dz} = $ ....

(a) $\frac{e^{x}}{\sin x}$

(b) $e^{x} \tan x$

(c) $e^{x} \cos x$

(d) $\frac{e^{x}}{\cos x}$

15 If $f(x) = 2x^3 - 3x^2 - 36x + 14$

Answer one of the following items :

[a] Find the local maximum and local minimum values of the function f

[b] Find the convexity intervals upwards and downwards of the function f

16 Find the area of the region bounded by the curve $y = 3x^2 + 4$, x-axis and the two straight lines $x = -1$, $x = 2$

17 The slope of the tangent to the curve at any point on it (x , y) is given by the relation $\frac{dy}{dx} = \sin x \cos x$, find the equation of the curve known that it passes through the point $\left(\frac{\pi}{6} , 1\right)$

18 A trapezium is drawn in a semi-circle , and its base is the diameter of the semi-circle , determine the base angle of the trapezium such that its area is as maximum as possible.

Model 4

Answer the following questions :

1 The equation of the normal to the curve $y = f(x)$ at the point $(1, 1)$ is $x + 4y = 5$, then $f'(1) = \dots\dots$

(a) -3
(b) $-\frac{1}{4}$
(c) 4
(d) -4

2 $\int \tan^2 x \, dx = \dots\dots$

(a) $\tan x - x + c$
(b) $\tan x + x + c$
(c) $\sec^4 x + c$
(d) $\frac{1}{3} \tan^3 x + c$

3 If $y = \ln(\sec x + \tan x)$, then $\frac{dy}{dx} = \dots\dots$

(a) $\tan x$
(b) $\sec x$
(c) $\tan^2 x$
(d) $\csc x$

4 If $z = x + \frac{1}{x}$, then $dz = \dots\dots$

(a) $\left(1 + \frac{1}{x^2}\right) dx$
(b) $\left(1 + \frac{1}{x^2}\right) dx + c$
(c) $\left(1 - \frac{1}{x^2}\right) dx + c$
(d) $\left(1 - \frac{1}{x^2}\right) dx$

5 $\int_{-2}^{2} (a x^3 + b x + c)\, dx$ depends on ....

(a) the value of b

(b) the value of c

(c) the value of a

(d) the value of a , b

6 $\lim\limits_{x \to 1} \frac{\ln x}{x-1} =$ ....

(a) zero

(b) 1

(c) e

(d) e^{-1}

7 If $y^2 = 1 - \frac{1}{x^2}$, **prove that :** $y \frac{d^2 y}{d x^2} + \left(\frac{dy}{dx}\right)^2 + \frac{3}{x^4} = 0$

8 If $f'(x) = x \cdot f(x)$, $f(3) = -5$

, then $f''(3) =$ ....

(a) -50

(b) -40

(c) 15

(d) 27

9 Find the absolute extrema values of the function $f : f(X) = X e^{-X}$ in the interval $[0, 2]$

10 If the function $f : f(X) = X^3 - 3X^2$

Answer one of the following items :

[a] Find the increasing and decreasing intervals of the function f

[b] Find the local maximum and local minimum values of the function f

11 If $x^2 y^3 = 108$, and $\frac{dx}{dt} = 2$ at $x = 2$, $y = 3$

, then $\frac{dy}{dt} = \dots\dots\dots\dots$

(a) 2

(b) -2

(c) -1

(d) 18

12 *Answer one of the following items :*

[a] Find : $\int (x^2 + 3)\sqrt{x-2}\, dx$

[b] Find : $\int e^{(x^3 + 2 \ln x)}\, dx$

13 The function $f : f(x) = x^3 - 12x$ is decreasing in the interval $\dots\dots\dots\dots$

(a) $]-2 , 2[$

(b) $\mathbb{R} - [-2 , 2]$

(c) $\mathbb{R} - \{2 , -2\}$

(d) $\mathbb{R} -]-2 , 2[$

14 Find the volume of a solid generated by revolution of the region bounded by the curve $x\,y = 3$ and the two straight lines $y = 1$, $y = 3$ and y-axis a complete revolution about x-axis.

15 Find the dimensions of a rectangle which has the greatest area can be drawn in a triangle , its base length 16 cm. and its height 12 cm. such that one of its sides coincides with the base of the triangle and its opposite vertices lie on the other two sides of the triangle.

16 The area of the region bounded by the curve $y = \sqrt{4 - x^2}$ and x-axis in square unit equals ....

(a) 2

(b) 4

(c) 2π

(d) 4π

17 Find the equation of the curve passes through the point A (2 , 3) and the slope of the normal at any point on it is $3 - x$

18 A regular quadrilateral pyramid of metal expands uniformaly , the height equals the side length of its base , its volume increases at a rate 1 cm.^{3}/sec. , if the rate of increasing of each of its height and its base side equals 0.01 cm./sec. find the base length.

Model 5

Answer the following questions :

1 $\lim\limits_{x \to 0} \left(\frac{\ln(1 + 2x)}{e^x - 1}\right) = \dots\dots\dots\dots$

(a) zero

(b) 1

(c) 2

(d) $\frac{1}{2}$

2 The function $f : f(x) = -|x| + 1$ is decreasing in the interval $\dots\dots\dots\dots$

(a) $]0, \infty[$

(b) $]-\infty, 0[$

(c) $]1, \infty[$

(d) $]-\infty, 0[$

3 The straight line $y + x - 1 = 0$ touches the curve of the function $f : f(x) = x^2 - 3x + a$, then $a = \dots\dots\dots\dots$

(a) 1

(b) 2

(c) 3

(d) 4

4 If $\int_{-2}^{3} f(x)\, dx = 12$, $\int_{-2}^{5} f(x)\, dx = 16$, then : $\int_{3}^{5} f(x)\, dx = \dots\dots\dots\dots$

(a) -28

(b) -4

(c) 4

(d) 28

5 Find the local maximum and local minimum value of the function f, where : $f(x) = x^4 - 2x^2$

6 $\int (2x - 1) e^{2x+3} \, dx = y z - \int z \, dy$, then : $\int z \, dy = \dots\dots\dots$

(a) $e^{2x+3} + c$

(b) $\frac{1}{2} e^{2x+3} + c$

(c) $- e^{2x+3} + c$

(d) $-\frac{1}{2} e^{2x+3} + c$

7 **Find :** $\int 3x \sqrt[3]{3x+1} \, dx$

8 If the function f : where $f(x) = \frac{x^2 + 9}{x}$

Answer one of the following items :

[a] Find the convexity intervals upwards and downwards of the curve of the function f

[b] Find the absolute maximum and absolute minimum values of the function f where $x \in [1 , 6]$.

9 ***Answer one of the following items :***

[a] Find : $\int \sec^5 x \tan x \, dx$

[b] Find : $\int x^2 \sec^2 (x^3 + 5) \, dx$

10 Find the area of the region bounded by the curve $y = 6 - x^2$, and the straight line $y = -x$

11 If $y = x^{n+1} + n\, x^{n-1} + 1$, then : $\frac{d^n y}{d x^n} = \dots\dots$

(a) $\lfloor n+1$

(b) $x \lfloor n+1$

(c) $x \lfloor n$

(d) $x^{-1} \lfloor n$

12 $\frac{d^3}{d x^3} (\sin^2 x) = \dots\dots$

(a) $\sin 2x$

(b) $2 \cos 2x$

(c) $4 \cos 2x$

(d) $-4 \sin 2x$

13 When the region bounded by the curve $x = \frac{1}{\sqrt{y}}$, $1 \le y \le 4$ and y-axis revolves a complete revolution about y-axis , then the volume of the solid generated in cubic units equals $\dots\dots$

(a) $\frac{2}{3} \pi$

(b) $3\sqrt{2}\, \pi$

(c) $2 \pi \ln 2$

(d) $\frac{2}{3} \pi \log 3$

14 On the perpendicular coordinate system a straight line $\overleftrightarrow{AB}$ passes through the point C (3 , 2) and intersects the positive part of X-axis at the point A and the positive part of y-axis at the point B , prove that the smallest area of the triangle AOB equals 12 square unit where O is the origin.

15 A ladder of length two metres is leaning against a smooth vertical wall. If the top of the ladder slid down at the same rate as the lower end slid away from the wall , then the distance of the lower end from the wall equals .... m.

(a) 2

(b) $2\sqrt{2}$

(c) $\sqrt{2}$

(d) $-\sqrt{2}$

16 The slope of the tangent to the curve of the function f at any point on it (X , y) is given by the relation $g(X) = \frac{X e^{X}}{(X+1)^2}$, find the equation of the curve given that the curve passes through (1 , 2 e)

17 The slope of the normal to the curve : $x = \cos\theta$, $y = \sqrt{2} + \sin\theta$ at $\theta = \frac{\pi}{4}$ is ....

ⓐ 1

ⓑ -1

ⓒ zero

ⓓ undefined

18 A regular hexagonal like lamina shrinks by cooling. The rate of change of its side length is 0.1 cm./sec. , find the rate of change in the area of the lamina when its side length is 10 cm.

Model 6

Answer the following questions :

1 The rate of change of tangent slope of the function $f : f(x) = 2x^3$ at $x = 3$ equals ....

(a) 54

(b) 36

(c) 18

(d) 9

2 The function $f : f(x) = x^x$ has a stationary point at $x =$ ....

(a) e

(b) $\frac{1}{e}$

(c) 1

(d) $\sqrt{e}$

3 $\frac{d}{dx} \int_2^3 x\sqrt{x^2+1}\, dx =$ ....

(a) -1

(b) zero

(c) 1

(d) 2

4 The slope of the tangent to the curve of the function $y = f(x)$ at a particular point is $\frac{1}{2}$ and the x-coordinate of this point decreases at a rate 3 units/sec. , then the rate of change of its y-coordinate equals .... unit/sec.

(a) $-\frac{1}{6}$

(b) $-\frac{3}{2}$

(c) $\frac{1}{6}$

(d) $\frac{3}{2}$

5 Find the nearest point to the point $(0 , 5)$ and lies on the curve $y = \frac{1}{2} x^2 - 4$

6 If $y = \sin^3 \theta$, $z = \cos^3 \theta$, then $\frac{dy}{dz} = \dots\dots\dots\dots$

(a) $-\sin \theta$

(b) $\cos \theta$

(c) $-\tan \theta$

(d) $3 \sin 2\theta$

7 The slope of the tangent to the curve at a point (x , y) which lies on it is $x\sqrt{x+1}$, find the equation of the curve given that it passes through $\left(0 , \frac{11}{15}\right)$

8 If $y = a(1 - \cos\theta)$, $x = a(\theta + \sin\theta)$

, then $\frac{dy}{dx} =$ ....

(a) $\tan\theta$

(b) $\cot\theta$

(c) $\tan\frac{\theta}{2}$

(d) $\cot\frac{\theta}{2}$

9 Find the volume of the solid generated by revolving the region bounded by the curve $y = \sqrt{8x}$ and the two straight lines $y = 6 - x$, $y = 0$ a complete revolution about x-axis.

10 Find the local maximum and local minimum values of the curve :

$y = \sin x\,(1 + \cos x)$ where $x \in \left]0\ , \frac{\pi}{2}\right[$

11 If $y = \pi^{\sin x} + e^{\pi}$, then $\frac{dy}{dx} = \dots\dots$

(a) $\pi^{\cos x}$

(b) $\sin x \times \pi^{\sin x - 1}$

(c) $\pi^{\sin x} \cos x$

(d) $\pi^{\sin x} \cos x \ln \pi$

12 ***Answer one of the following items :***

[a] Find : $\int \ln x \, dx$

[b] Find : $\int 2 \cos^2 x \, dx$

13 If the function $f : f(x) = x^2 + \frac{b}{x}$ has a critical point at $x = 2$

, then b = $\dots\dots$

(a) 16

(b) 4

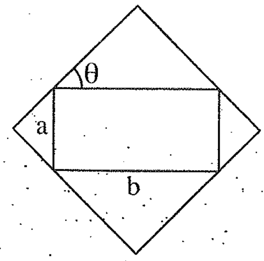
(c) -1

(d) $\frac{1}{16}$

14 The gas leaks from a spherical balloon at a rate x cm³/sec. , **prove that :** the rate of decreasing of the balloon external surface area of the moment its radius r cm. equals $\frac{2x}{r}$ cm²/sec.

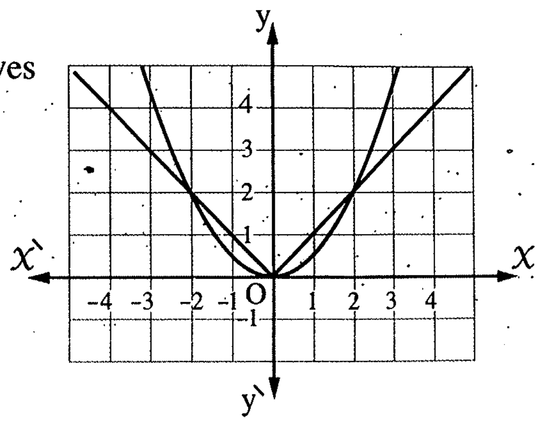
15 **In the opposite figure :**

Find the greatest area of a rectangle that can be drawn outside a rectangle whose dimensions are constants a and b



16 **In the opposite figure :**

The area of the region bounded by the two curves $y = x^2$, $y = |x|$ equals ....



(a) $2 \int_{-1}^{0} (x^2 - x)\, dx$

(b) $\int_{0}^{1} (x - x^2)\, dx$

(c) $2 \int_{0}^{1} (x - x^2)\, dx$

(d) $\int_{-1}^{1} (x - x^2)\, dx$

17 If $y = \ln \sqrt{\tan x}$, then $\frac{dy}{dx} =$ .... at $x = \frac{\pi}{4}$

(a) 1

(b) zero

(c) $\frac{1}{2}$

(d) ∞

18 ***Answer one of the following items :***

[a] If $x^2 + y^2 = 1$, **prove that :** $y^3 \frac{d^2 y}{dx^2} + 1 = 0$

[b] If $y = \cos x$, **prove that :** $\left(\frac{dy}{dx}\right)^2 - y \frac{d^2 y}{dx^2} = 1$

Model 7

Answer the following questions :

1. The tangent to the curve of the function $y = \sqrt[3]{X}$ at $X = 0$ is parallel to ....
   - (a) X-axis.
   - (b) y-axis.
   - (c) the straight line $y = X$
   - (d) $X + y = 0$

2. If $\int_1^3 f(X)\, dX = 5$, $\int_3^4 f(X)\, dX = 2$, $\int_2^4 f(X)\, dX = 6$
   , then $\int_2^1 f(X)\, dX =$ ....
   - (a) 1
   - (b) 13
   - (c) -2
   - (d) -1

3. $\lim\limits_{X \to 6} \dfrac{e^X - e^6}{X - 6} =$ ....
   - (a) 1
   - (b) -1
   - (c) zero
   - (d) e^6

4. The maximum value of the function $y = \dfrac{\ln X}{X}$ in the interval $[2 , \infty[$ is ....
   - (a) 1
   - (b) $\dfrac{2}{e}$
   - (c) e
   - (d) $\dfrac{1}{e}$

5 Determine the convexity intervals upward and downward of the curve of the function $f : f(x) = \frac{x^2 + 1}{x^2 + 3}$ also find the inflection points if existed.

6 If $f''(x) = \frac{1}{2}\left[e^{x} + e^{-x}\right]$, $f(0) = 1$, $f'(0) = 0$, then $f(x) = \ldots\ldots$

(a) $-f'(x)$

(b) $f'(x)$

(c) $-f''(x)$

(d) $f''(x)$

7 If $x^2 y = 2x + 5$, **prove that :** $x^2 \frac{d^2 y}{d x^2} + 4x \frac{d y}{d x} + 2y = 0$

8 When the region bounded by the curve $y = x^2$ and the straight line $y = 2$ revolves a complete revolution about y-axis , then the volume of the generated solid equals ....

(a) $\int_0^2 y \, dx$

(b) $\pi \int_0^2 y \, dy$

(c) $\pi \int_0^2 x \, dx$

(d) $\pi \int_0^2 x^2 \, dx$

9 An equilateral triangle , its side length increase at a rate $\frac{1}{3}$ cm./sec. , then the rate of change of its perimeter at this instant equals .... cm./sec.

(a) 1

(b) 2

(c) 3

(d) 4

10 If $x = 2t^3 + 3$, $y = t^4$, then $\frac{d^2 y}{d x^2} =$ .... at $t = 1$

(a) 9

(b) $\frac{1}{9}$

(c) 3

(d) $\frac{1}{3}$

11 ***Answer one of the following items :***

[a] Find : $\int \cot^3 x \, dx$

[b] Find : $\int x^3 \ln x \, dx$

12 The function $f : f(X) = X^3 + 4X + 2$ is increasing for every $X \in$ ....

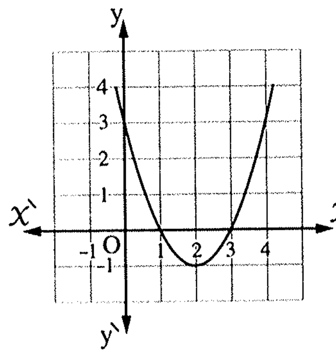
(a) $\mathbb{R}^+$

(b) $\mathbb{R}$

(c) $\mathbb{R}^-$

(d) $\mathbb{R} - \{0\}$

13 The opposite figure represents the curve $f'(X)$, then the function f has a local minimum at $X =$ ....



(a) 1

(b) 2

(c) 3

(d) 4

14 A metalic circular sector whose area is 16 cm^2. Find the radius length of the sector's circle which makes its perimeter as minimum as possible. What is the measure of its angle then ?

15 *Answer one of the following items :*

[a] Find the area of the region bounded by the two curves : $y = 2x^2$, $y = 3 - x^2$

[b] Find the area of the region bounded by the two curves : $y = x^2$, $y = 2 - x$

16 The slope of the normal to the curve of any point (x , y) on it equals $\frac{5 + 2y}{2 - 3x^2}$ and the curve passes through $(1 , 2)$ find its equation.

17 **Find the equation of the tangent and normal to the curve :**

$2 + \ln y \,.\, \ln x = x^2 + y$ at the point whose x coordinate is 1

18 A 180 cm. man moves far from the base of a 3-metre lamp post at a rate of 1.2 m./s. , find the rate of change of the length of the man's shadow. If the straight line passing through the highest point of the man's head and the top of the lamp inclines on the ground with an angle of measure θ^{rad} when the man is far from the base of the lamp post for a distance x metre , **prove that :** $x = \frac{6}{5} \cot \theta$, then find the rate of change of θ when the man is 3.6 m. far from the base of the lamp post.

Model 8

Answer the following questions :

1 The function $f : f(\mathcal{X}) = \mathcal{X}^3 + 4\mathcal{X} + 2$ is increasing at $\mathcal{X} \in$ ....

(a) $]-4, \infty[$

(b) $\mathbb{R}$

(c) $]-\infty, \frac{-4}{3}[$

(d) $]\frac{-4}{3}, \infty[$

2 The tangent equation of the curve of the function f :

$f(\mathcal{X}) = e^{2\mathcal{X}+1}$ at the point $\left(-\frac{1}{2}, 1\right)$ is ....

(a) $2y = \mathcal{X} + 1$

(b) $y = 2\mathcal{X} + 2$

(c) $y = 2\mathcal{X} - 3$

(d) $2y = 3\mathcal{X} + 1$

3 $\int_0^{10\pi} |\sin \mathcal{X}| \, d\mathcal{X} =$ ....

(a) 10

(b) 10π

(c) 20

(d) 20π

4 Determine the absolute minimum value and absolute maximum value of the function f , where $f(\mathcal{X}) = \mathcal{X} + \frac{1}{\mathcal{X}}$ in the interval $\left[\frac{1}{2}, 3\right]$

5 A regular octagon , its side length is 10 cm. the side length increase at a rate 0.2 cm./sec. Find the rate of increasing of its area.

6 $\lim\limits_{x \to 0} \frac{a^x + b^x + c^x - 3}{x} = \dots\dots$

(a) ln (a + b + c)

(b) ln (a b c)

(c) ln a – ln b – ln c

(d) log (a b c)

7 The normal equation to the curve $y = f(x)$ at the point (1 , 1) is $x + 4y = 5$, then $f'(1) = \dots\dots$

(a) -3

(b) $-\frac{1}{4}$

(c) 4

(d) -4

8 The radius length of a circle increases at a rate 2 cm./min. and its area of a rate 20π cm^2/min. , then the radius length at this instant equals …… cm.

(a) $\frac{5}{2}$

(b) 5

(c) 10

(d) 20

9 If y = ln (sec x + tan x) , then $\frac{dy}{dx}$ = ...

(a) tan x

(b) sec x

(c) $\tan^2 x$

(d) csc x

10 If the function f : where $f(x) = 1 + 6x - 2x^3$

Answer one of the following items :

[a] Find the local maximum and local minimum values of the function f

[b] Find the convexity intervals upwards and downwards and the inflection points (if existed) of the curve of the function f

11 ***Answer one of the following items :***

[a] Find : $\int (1+4x^4)\, e^{x^4}\, dx$

[b] Find : $\int \frac{x}{\sqrt{x+2}}\, dx$

12 The length of the hypotenuse in a right-angled triangle equals 10 cm. , find the length of each side of the right angle when the area is as large as possible.

13 The slope of the tangent at any point (x , y) on the curve $y = f(x)$ is : $6x^2 - 30x + 36$, find the curve equation given that the curve has a local maximum value equals 28

14 The area of the region bounded by the two curves $y = x^2$, $y = x^3$ is .... square unit.

(a) 1

(b) $\frac{7}{12}$

(c) $\frac{1}{12}$

(d) 2

15 If $y = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots$ to ∞ , then $2\,y'' + 3\,y' - 4\,y =$ ....

(a) y

(b) zero

(c) $2\,y$

(d) $9\,y$

16 If $y = e^{x} \sin x$, **prove that :** $\frac{d^2 y}{d x^2} - 2\frac{d y}{d x} + 2\,y = 0$

17 $\int \frac{\sin^6 x}{\cos^8 x}\, d\,x =$ ....

(a) $\tan^7 x + c$

(b) $\frac{1}{7} \tan^7 x + c$

(c) $\frac{1}{7} \tan 7\,x + c$

(d) $\sec^7 x + c$

18 Find the volume of the solid generated by revolving the plane region bounded from the top by the curve $x^2 + y^2 = 4$ and from the bottom by the two stright lines $y = x$, $y = -x$ a complete revolution about x-axis

Model 9

Answer the following questions :

1 If $\int_{1}^{4} f(x)\, dx + \int_{2b}^{8} f(x)\, dx = \int_{1}^{8} f(x)\, dx$, then b = ...

(a) 2

(b) 4

(c) 1

(d) 8

2 If y = a cos (ln x) + b sin (ln x) , **prove that :** $x^2 \frac{d^2 y}{d x^2} + x \frac{d y}{d x} + y = 0$

3 $\lim_{x \to 0} (1 + 3 \tan^2 x)^{\cot^2 x} =$ ....

(a) e

(b) e^3

(c) 3 e

(d) $e^{\frac{1}{3}}$

4 If $x > 0$, then the smallest value of the expression $x + \frac{1}{x}$ equals ....

(a) zero

(b) 1

(c) 2

(d) 4

5 The slope of the tangent to the curve of the function f at any point (x , y) on it is given by the relation $g(x) = x e^{3x}$, find the curve equation given that it passes through the point $\left(\frac{1}{3} , 5\right)$

6 The ratio between the slope of the tangent to the curve $y = \ln 3\sqrt{x+1}$ and the slope of the tangent to the curve $y = \ln 5\sqrt{x+1}$ at $x = a$ is ....

(a) 3 : 5

(b) 5 : 3

(c) 1 : 1

(d) ln 3 : ln 5

7 If $x = \ln t$, $y = \sin t$, then $\frac{dy}{dx} = \dots\dots$

(a) $\cos t$

(b) $t \cos t$

(c) $t^2 \sin t$

(d) $t^2 \cos t$

8 The length of each of two equal sides in an isosceles triangle equals 6 cm. and the measure of the angle between them equals (x) , the rate of change of x is $\left(\frac{\pi}{90}\right)$ per minute then the rate of change of its area at $x = 30°$ is $\dots\dots$

(a) $\frac{\sqrt{3}}{10}\pi$

(b) $\frac{\pi}{10}$

(c) $9\sqrt{3}$

(d) 9

9 If $f(x) = \ln\left(x + \sqrt{x^2+1}\right)$, then $\acute{f}(x) = \dots\dots$

(a) $\sqrt{x^2+1}$

(b) $\frac{x}{\sqrt{x^2+1}}$

(c) $1 + \frac{x}{\sqrt{x^2+1}}$

(d) $\frac{1}{\sqrt{x^2+1}}$

10 ***Answer one of the following items :***

[a] Find : $\int \sqrt{\frac{1+\sqrt{x}}{x}}\, dx$

[b] Find : $\int (\sin^2 x + \cos^2 x + \cot^2 x)\, dx$

11 If the function $f : f(X) = X^3 - 3X + 4$

Answer one of the following items :

[a] Find the increasing and decreasing intervals to the curve of the function.

[b] Sketch the curve of the function , show the points of local maximum and local minimum and the inflection points (if existed).

12 The normal equation to the curve $y = x\,|x|$ at the point $(-2\ ,-4)$ is ....

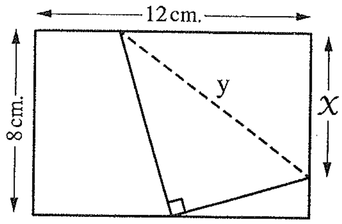
(a) $y + 4x + 12 = 0$

(b) $4y + x + 18 = 0$

(c) $4y + x + 14 = 0$

(d) $y + 4x - 4 = 0$

13 The top right corner of a piece of paper whose dimensions are 8 cm. , 12 cm. is folded to the lower edge as shown in the figure. What is the value of x which makes y as small as possible ?



14 $\pi \int_{-2}^{2} (4 - x^2)\, dx$, is the volume of ....

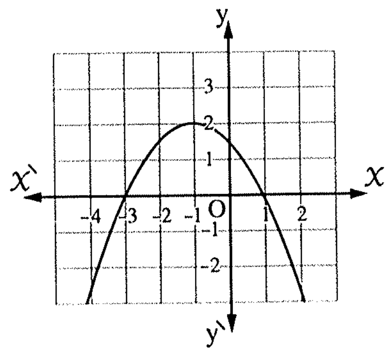
(a) A sphere whose radius length is 4 units.

(b) A right circular cone whose height is 4 units.

(c) A sphere whose radius length is 2 units.

(d) A right circular cylinder whose height is 4 units.

15 The opposite figure represents the curve $f'(x)$ of the function f , then the solution set of the inequality $f''(x) > 0$ is ....



(a) $]-1 , \infty[$

(b) $]1 , \infty[$

(c) $]-\infty , -1[$

(d) $]-\infty , 1[$

16 If $y = \sin(\csc 3x^2) + \sin(5x^3) \csc(5x^3)$. **Find :** $\frac{dy}{dx}$

17 **Find the area of the region bounded by the curve :**

$y = x^2 - 9$, the x-axis , the straight line $x = 4$ and above x-axis.

18 The gas leaks from a spherical balloon at a rate 20 cm^3/sec. Find the rate of change of the balloon exteranal surface area at the moment which the radius length is 10 cm.

Model 10

Answer the following questions :

1 The curve $y - e^{xy} + x = 0$ has a vertical tangent at the point ....

ⓐ $(1 , 1)$

ⓑ $(0 , 0)$

ⓒ $(1 , 0)$

ⓓ $(2 , e^2)$

2 $\lim\limits_{x \to 0} \dfrac{(10)^{\sin x} - 1}{\tan x} =$ ....

ⓐ log 10

ⓑ ln 10

ⓒ ln sin x

ⓓ 1

3 $\int_0^6 |3 - x| \, dx =$ ....

ⓐ 9

ⓑ -9

ⓒ $\frac{9}{2}$

ⓓ $\frac{-9}{2}$

4 Determine the absolute maximum value and absolute minimum value of the function

$f : f(x) = \dfrac{x}{x^2 + 1}$, $x \in [0 , 2]$

5 The height of a cylinder which has the greatest volume placed inside a sphere whose radius length (r) equals ....

(a) $\frac{2r}{\sqrt{5}}$

(b) $\frac{2r}{\sqrt{3}}$

(c) $2r$

(d) $2\sqrt{3}\,r$

6 If $\frac{dy}{dx} = 7 - \sin 2x$ find y in terms of x if $y = 5$, $x = 0$

7 A point moves on a curve whose equation is $x^2 + y^2 - 4x + 8y - 6 = 0$, the rate of change of its x-coordinate with respect to time at point (3 , 1) equals 4 units/sec. , then the rate of change of its y-coordinate with respect to time t at the same point equals ....

(a) $\frac{3}{5}$

(b) $\frac{-4}{5}$

(c) $\frac{-3}{5}$

(d) $\frac{4}{5}$

8 If $y = \ln\sqrt{\tan x}$, then $\frac{dy}{dx} =$ .... when $x = \frac{\pi}{4}$

(a) 1

(b) zero

(c) $\frac{1}{2}$

(d) ∞

9 Sketch the curve of the function f where $f(x) = x^3 - 3x + 2$

10 ***Answer one of the following items :***

[a] Find : $\int \frac{\ln x}{x} \, dx$

[b] Find : $\int x (x^2 + 3)^5 \, dx$

11 *Answer one of the following items :*

[a] If $x = a \sec^2 \theta$, $y = a \tan^3 \theta$, **prove that :** $\frac{d^2 y}{d x^2} = \frac{3 \cot \theta}{4 a}$

[b] If $y = x \tan x$, **prove that :** $\frac{d^2 y}{d x^2} = 2 (1 + y) \sec^2 x$

12 The current intensity I (Ampere) in a circuit for alternating current at any moment t (second) is given by the relation $I = 2 \cos t + 2 \sin t$, what is the maximum value of the current in this circuit ?

13 A ladder of constant length its upper end slides on a vertical wall at a rate k unit/sec. **Find :** the rate of increasing of the distance between the lower end and the wall when the ladder inclined to the vertical with an angle θ where $\csc \theta = \frac{5}{4}$

14 $\int \frac{e^{-x} - 1}{e^{-x} + x} \, dx = \dots\dots\dots\dots$

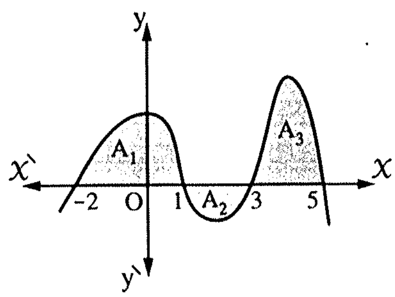
(a) $-\ln |e^{x} + x| + c$

(b) $\ln |e^{-x} + x| + c$

(c) $\ln |e^{x} + x| + c$

(d) $-\ln |e^{-x} + x| + c$

15 **In the opposite figure :**



If $A_1 = 5$ square unit

, $A_2 = 2$ square unit

, $A_3 = 8$ square unit

, then $\int_{-2}^{5} f(x) \, dx + \int_{-2}^{5} |f(x)| \, dx = \dots\dots\dots\dots$

(a) 15

(b) 20

(c) 22

(d) 26

16 If $f(x) = x - x \ln x$, then the slope of the tangent to the curve at $x = e$ equals ....

(a) zero

(b) -1

(c) 1

(d) e

17 Find the volume of the solid generated by revolving the plane region bounded by the curve $y = 2\sqrt{x-1}$ (where $x \geq 1$) and the tangent at the point (2 , 2) and the straight line $y = 0$ a complete revolution about x-axis

18 If the normal to the curve $y = x \ln x$ is parallel to the straight line $2x - 2y + 3 = 0$, then the normal equation is ....

(a) $x - y = 3e^{-2}$

(b) $x - y = 6e^{-2}$

(c) $x - y = 3e^{2}$

(d) $x - y = 6e^{2}$

Model 11

Answer the following questions :

1 $\lim\limits_{x \to \infty} \left(\frac{x+5}{x+3}\right)^{x} =$ ....

(a) e

(b) e^2

(c) $\frac{1}{e}$

(d) $\frac{2}{e}$

2 The rate of change of tangent slope of a curve at any point (x , y) on it is $6(1 - 2x)$ and the curve has a critical point at $x = 1$ and the function has a local minimum value equals 4 . Find the normal equation to the curve at $x = -1$

3 The measure of the angle which the tangent to the curve $\sin 2X = \tan y$ makes with the positive direction of X-axis at the point $\left(\frac{3\pi}{4}, \frac{3\pi}{4}\right)$ equals ....

(a) zero

(b) 135°

(c) 45°

(d) 26° 34`

4 If $y = X^2 + 3X + 2$, $z = 3X^2 - 5X + 4$

, then $\frac{d^2 y}{d z^2}$ at $X = 2$ equals ....

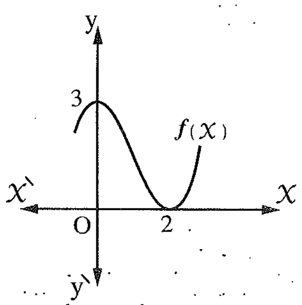
(a) $\frac{-4}{7}$

(b) $\frac{-4}{49}$

(c) 8

(d) 56

5 **In the opposite figure :** $\int_0^2 [f(X)]^2 f`(X)\, dX =$ ....



(a) − 9

(b) 9

(c) 2

(d) 1

6 A rectangle whose perimeter is 40 m. , its area is as great as possible when its dimensions are .... m.

(a) 15 , 5

(b) 10 , 10

(c) 12.5 , 7.5

(d) 9 , 11

7 If $f(x) = (x + 1)(x^2 - x - 2)$

Answer one of the following items :

[a] Find the local maximum and local minimum values of the function f

[b] Find the inflection points of the curve of the function (if existed) , then sketch the curve of the function.

8 ABC is right-angled triangle at $\angle C$, its area is constant and equals 24 cm^2. the rate of change of b equals 1 cm./sec. Find the rate of change of a and m ($\angle A$) at the instant when b equals 8 cm.

9 If $y = \sec x$, **prove that :** $y \frac{d^2 y}{d x^2} + \left(\frac{d y}{d x}\right)^2 = y^2 (3 y^2 - 2)$

10 If $f(X) = \begin{cases} 2X^3 + 3 & , X \leq -1 \\ 3X + 4 & , X > -1 \end{cases}$, then $\int_{-2}^{2} f(X)\, dX = \dots\dots$

(a) 6
(b) 9
(c) 12
(d) 15

11 Find the area of the plane region bounded by the curve $y = X^2 - 9$ and X-axis and the straight line $X = 4$

12 The normal equation to the curve $y = 3e^X$ at the point which lies on the curve and its X-coordinate is -1 is $\dots\dots$

(a) $e\,y = 3X$
(b) $3X + e\,y + 6 = 0$
(c) $y - eX - 4e = 0$
(d) $e^2 X + 3e\,y - 9 + e^2 = 0$

13 If f is a differentiable odd function in the interval $]-\infty, \infty[$ and $f'(3) = 2$, then $f'(-3) = \dots\dots$

(a) zero
(b) 1
(c) 2
(d) 4

14 If $y = \ln(\sin x)$, then : $\frac{d^2 y}{d x^2} =$ ....

(a) $-\csc^2 x$

(b) $\sec x$

(c) $-\csc x \cot x$

(d) $\sec x \tan x$

15 ***Answer one of the following items :***

[a] Find : $\int \frac{\cos^2 x}{1 - \sin x} \, d x$

[b] Find : $\int \frac{1}{x \ln x^3} \, d x$

16 If $x \in [0 , \pi]$, then the function $f : f(x) = x \sin x + \cos x$ has an absolute minimum value at $x =$ ....

(a) zero

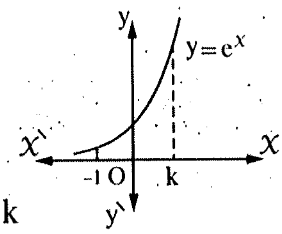
(b) $\frac{\pi}{2}$

(c) π

(d) -1

17. A cuboid with square base , the sum of all its edges is 240 cm. Find the dimensions of the cuboid when its volume is maximum.

18. **In the opposite figure :**



The volume of a solid generated by revolving the shaded region a complete revolution about X-axis and the straight line $X = -1$, $X = k$ equals $\frac{\pi}{2}(e^{10} - e^{-2})$ cube unite. Find the value of k

Model 12

Answer the following questions :

1 If $\int_{3}^{5} f(X)\, dX = 6$, then $\int_{3}^{5} 4 f(X) - 1\, dX =$ ....

ⓐ 18

ⓑ 22

ⓒ 23

ⓓ 26

2 If $f(X) = e^{\tan X}$, then $\lim\limits_{X \to \frac{\pi}{4}} \dfrac{f(X) - f\left(\frac{\pi}{4}\right)}{X - \frac{\pi}{4}} =$ ....

ⓐ e

ⓑ 2 e

ⓒ e^2

ⓓ $2\, e^2$

3 The normal equation to the curve $y = \sin X$ at $(0 , 0)$ is ....

ⓐ $X = 0$

ⓑ $y = 0$

ⓒ $X + y = 0$

ⓓ $X - y = 0$

4 The minimum value of the function $f : f(X) = X \ln X$ equals ....

ⓐ e

ⓑ $\frac{1}{e}$

ⓒ $\frac{-1}{e}$

ⓓ $-$ e

5 Find the curve equation $y = f(X)$ if $\frac{d^2 y}{d X^2} = a X + b$ where a , b are constant and the curve has an inflection point (0 , 2) and local minimum value at the point (1 , 0)

6 Find the local maximum value and the local minimum value of the function :

$y = \frac{1}{3} x^3 - 9x + 2$

7 If $e^{xy} = x^2 + y$ **Prove that :** $(x e^{xy} - 1)\, \acute{y} = 2x - y e^{xy}$

8 If $f(x) = x^{2019}$, then the 2019^{th} derivative of this function equals ....

(a) 2019

(b) $\lfloor 2018$

(c) $\lfloor 2019$

(d) zero

9 ***Answer one of the following items :***

[a] Find : $\int e^{x}(\cot x - \csc^2 x)\, dx$

[b] Find : $\int (13)^{x}\, dx$

10 A 5-metre rod is fixed by a hinge to the ground at its base. If its top rises up by a winch at a rate of 1 m./min. , find the rate of decreasing the projector length of the rod on the ground when the height of the top is 3 metres.

11 If $f(x) = (\cos x)^{\cos x}$, then $f`(0) =$ ....

(a) -3

(b) -2

(c) -1

(d) zero

12 $\int_{-2}^{4} |x^2 - 3x| \, dx = \dots\dots$

(a) 9

(b) 15

(c) 18

(d) 24

13 The function $f : f(x) = x^3 + 4x + 8$ increases at $x \in \dots\dots$

(a) $]-4, \infty[$

(b) $]-\infty, \frac{-4}{3}[$

(c) $]\frac{-4}{3}, \infty[$

(d) $\mathbb{R}$

14 The area of the plane region bounded by the two curves $y^3 = x$, $y = x$ equals $\dots\dots$

(a) $\frac{1}{2}$

(b) $\frac{3}{4}$

(c) $\frac{-3}{4}$

(d) $\frac{-1}{2}$

15 If $y = \sec^n(x)$, then $\frac{dy}{dx} = \dots\dots$

(a) $n\, y \sec x$

(b) $n\, y \tan x$

(c) $n\, y \sec^2 x$

(d) $n\, y \tan^2 x$

16 A rectangle is drawn such that two adjacent vertices of the rectangle lie on the curve $y = x^2 - 12$ and the other two vertices lie on the curve $y = 12 - x^2$, find the maximum area of this rectangle.

17 Investigate the convexity of the curve of the function f where $f(X) = \sqrt[3]{X-3}$ and find the inflection points.

18 ***Answer one of the following items :***

[a] Find the volume of the solid generated by revolving the region bounded by the curve $y = \sqrt{X}$, and the straight line $y = X$ a complete revolution about X-axis.

[b] Find the volume of the solid generated by revolving the region bounded by the curve $y = \sqrt{X+5}$ and the straight lines $y = 0$, $X = 1$, $X = 3$ a complete revolution about X-axis.

Model 13

Answer the following questions :

1 If $y = \frac{z+1}{z-1}$, $X = \frac{z-1}{z+1}$, then $\frac{dy}{dX} =$ .... at $X = 2$

(a) -9

(b) 4

(c) $\frac{1}{4}$

(d) $-\frac{1}{4}$

2 The height of the right cone which can be placed inside a sphere whose radius length is 9 cm. such that its volume is as great as possible equals .... cm.

(a) 7

(b) 12

(c) 8

(d) 10

3 The tangent to the curve $X = e^{\theta} \cos \theta$, $y = e^{\theta} \sin \theta$ at the point at which $\theta = \frac{\pi}{4}$ makes with the positive X-axis an angle of measure ....

(a) zero

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

4 $\int_0^2 \sqrt{4-X^2}\, dX =$ ....

(a) zero

(b) 2

(c) π

(d) $\frac{\pi}{2}$

5 Determine the intervals of convexity upwards and downwards and inflection points if existed to the curve $y = x(x^2 - 1)$

6 $\int \frac{\sec^2 x}{\tan x} \, dx = \dots\dots\dots$

(a) $-\frac{1}{2} \tan^{-2} x + c$

(b) $\ln |\tan x| + c$

(c) $\ln |\sec^2 x| + c$

(d) $\frac{1}{3} \sec^3 x + c$

7 $\lim\limits_{x \to \infty} \left(\frac{x+4}{x-2}\right)^{x+3} = \dots\dots\dots$

(a) e^2

(b) e^{-6}

(c) $-e^2$

(d) e^6

8 ***Answer one of the following items :***

[a] Find : $\int (\sin x + \cot x)^9 (\cos x - \csc^2 x) \, dx$

[b] Find : $\int \frac{x - \frac{1}{2}}{\sqrt{2x-1}} \, dx$

9 If f is a function where $f(X) = X|X - 2|$

Answer one of the following items :

[a] Determine the increasing and decreasing intervals of the function f

[b] Calculate the absolute maximum and absolute minimum values of the function in the interval $\left[\frac{1}{2}, \frac{5}{2}\right]$

10 The greatest value of the expression $\left(\sin X + \sqrt{3} \cos X\right)$ is at $X =$ ....

(a) $\frac{\pi}{3}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$

(d) zero

11 In a closed electric circuit , V is the potential difference (Volt) , I is the current intensity (Ampere) R is the resistance (Ohm). If the potential difference icreases at a rate of 1 Volt/s. and the current intensity decreases at a rate of $\frac{1}{2}$ Ampere/s. Find the rate of the resistance at the moment which V = 12 Volt and I = 2 Amperes.

12 If $y = 2 \sin x - x \cos x$ **Prove that :** $\frac{d^2 y}{d x^2} + y = 2 \sin x$

13 If $\int (2x + 3) \ln x \, dx = y z - \int z \, dy$

, then y z = ...

(a) $2x \ln x$

(b) $(2x + 3) \ln x$

(c) $\frac{1}{2} (2x + 3) \ln x$

(d) $x (x + 3) \ln x$

14 If the area of the region bounded by the two curves $y = 2x^2$, $y^2 = 4ax$ equals $\frac{2}{3}$ square unit. Find the value of a where $a > 0$

15 If $f(x) = (a - 2)x^2 + 3x - 5$, $x \in \mathbb{R}$, then the curve of the function f is concave downwards when ....

(a) $a > 2$

(b) $a < 2$

(c) $a = 2$

(d) $a = 0$

16 If $f(x) = 2 \sin \frac{x}{2} \cos \frac{x}{2}$, then the thousandth derivative of this function equals ....

(a) $\sin x$

(b) $\cos x$

(c) $-\sin x$

(d) $-\cos x$

17 A factory is producing electric appliances with profit L.E. 50 in every appliance if it produces 80 appliances monthly. When the production increased than that , the profit of each appliance decreases by 50 piasters for every extra appliance produced. Find the number of appliances produced monthly if the profit is to be maximum.

18 Find the volume of the solid generated by revalving the plane region bounded by the curve $y = \frac{4}{x}$ and the straight line $y + x = 5$ a complete revolution about x-axis.

Model 14

Answer the following questions :

1 The slope of the tangent to the curve $x y^3 = 3$ at the point $(3, 1)$ equals ....

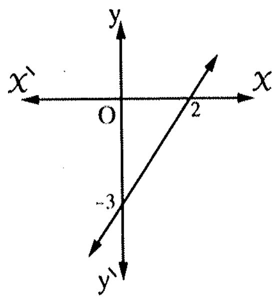
ⓐ $\frac{1}{9}$

ⓑ $-\frac{1}{9}$

ⓒ $\frac{2}{3}$

ⓓ $-\frac{2}{3}$

2 The opposite figure represents the first derivative of the function f and the function f has a local minimum equals -3 , then : $\int_0^3 f(x)\, dx =$ ....



ⓐ $\frac{27}{4}$

ⓑ -27

ⓒ $\frac{9}{4}$

ⓓ $-\frac{27}{4}$

3 The tangent to the curve $x = t^2 - 1$, $y = t^2 - t$ is parallel to x-axis at $t =$ ....

ⓐ zero

ⓑ $\frac{1}{\sqrt{3}}$

ⓒ $\frac{1}{2}$

ⓓ $-\frac{1}{\sqrt{3}}$

4 Find the equation of the curve which passes through the point $\left(\frac{\pi}{2}, \frac{\pi^2}{4} + 9\right)$ given that the slope of its tangent at any point on it (x, y) is given by the relation $m = 2x + \frac{1}{2}\sec^2\frac{x}{2}$

5 The function f is a differentiable function on $\mathbb{R}$,

$$f(x) = \begin{cases} 2x^2 + ax + b & , x \geq 1 \\ 3x - x^2 & , x < 1 \end{cases}$$

Find the value of a , b , then determine the convexity intervals upwards and downwards and the inflection points if existed.

6 $\lim_{x \to 0} \frac{e^{x} - e^{\sin x}}{x - \sin x} = \ldots\ldots\ldots\ldots$

(a) 1

(b) e

(c) zero

(d) – e

7 $\int_{-a}^{a} \frac{x}{x^4 + \cos x} \, dx = \ldots\ldots\ldots\ldots$

(a) – a

(b) 2 a

(c) zero

(d) $\frac{a^2}{a^5 + \sin a}$

8 ***Answer one of the following items :***

[a] Find : $\int \frac{x e^{x}}{(x+1)^2} \, dx$

[b] Find : $\int \frac{2x}{\sqrt{x^2}} \, dx$

9 The normal equation to the curve $2y = 3 - x^2$ at the point (1 , 1) is ....

(a) $x + y = 0$

(b) $x + y + 1 = 0$

(c) $x - y + 1 = 0$

(d) $x - y = 0$

10 The volume of the solid generated by revolving the region bounded by the curve $f(X) = X^2$ and X-axis and the two straight lines $X = -2$, $X = 2$ a complete revolution about X-axis equals ....

(a) $\frac{16\pi}{5}$

(b) $\frac{32\pi}{5}$

(c) $\frac{64\pi}{5}$

(d) 4π

11 A 3-metre wall is 3 metres away from a house , find the minimum length of the ladder that joined the ground and the house resting on the wall.

12 If $\sin X = X y$ **Prove that :** $X^2 (y + \overset{\backslash\backslash}{y}) + 2 \cos X = 2 y$

13 The local minimum value of the function $f : f(X) = X + \frac{1}{X}$ is at $X =$ ....

(a) -1

(b) zero

(c) 1

(d) 2

14 Find the area of the plane region under the curve $y = \sqrt{3X + 4}$ and above the X-axis. between the two straight lines $X = 0$, $X = 4$

15 If $y = x^{x}$, $x > 0$, then $\frac{dy}{dx} = \ldots\ldots\ldots\ldots$

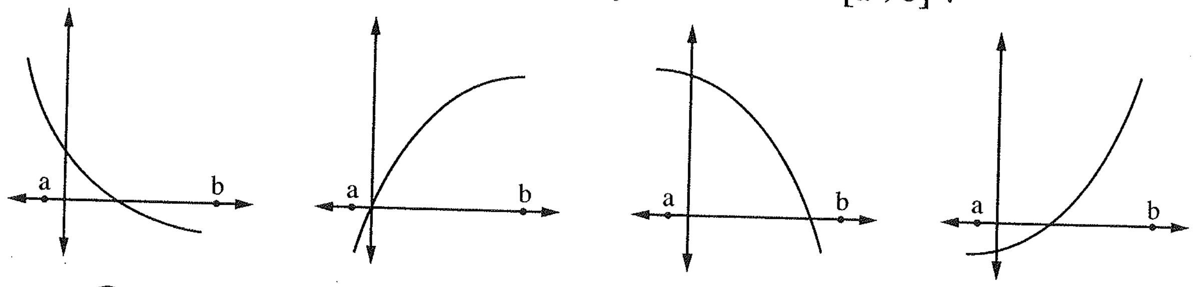
(a) $\ln x$

(b) $2 + \ln x$

(c) $x^{x} \ln x$

(d) $x^{x} (1 + \ln x)$

16 If $f'(x) < 0$, $f''(x) > 0$ for every $x \in [a , b]$, which of the following shown curves represents the curve of the function f in the interval $[a , b]$?



(a) (b) (c) (d)

17 Determine the increasing and decreasing intervals of the function $f : f(x) = 2 \ln x - x^{2}$

18 A cuboid whose dimensions at an instant are 3 , 4 , 12 cm. , the first dimension increases at a rate 2 cm./sec. , and the second dimension increases at a rate 1 cm./sec. and third dimension decreases at a rate 3 cm./sec.

Answer one of the following items :

[a] Find the rate of change of volume of the cuboid after 2 seconds.

[b] Find the rate of change of the diagonal length of the cuboid after 2 seconds.

Model 15

- **Answer the following questions :**

1 If $\int_{1}^{k} 3x^2 \, dx = 7$, then k = ...

(a) 2

(b) 1

(c) – 2

(d) – 1

2 Find the local maximum value and local minimum value of the function $f : f(x) = e^{x}(3 - x)$

3 If $f(\sin x) = \sin^2 x$, then $f'''(1)$ = ...

(a) 1

(b) 2

(c) π

(d) $\frac{\pi}{2}$

4 The curve $\left(\frac{x}{a}\right)^t + \left(\frac{y}{b}\right)^t = 2$ touches the straight line $\frac{x}{a} + \frac{y}{b} = 2$ at the point (a , b) when ....

(a) t = 3

(b) t = 2

(c) for all values of t

(d) not for any t

5 The normal equation to the curve $y = e^{2x} \cos x$ at $x = 0$ is ....

(a) $x + y = 2$

(b) $2y + x = 2$

(c) $2x + y = 2$

(d) $x - y = 2$

6 If $x = 4t$, $y = \frac{4}{t}$, then $\frac{dy}{dx} =$ .... at $t = 3$

(a) $\frac{1}{4}$

(b) $\frac{1}{16}$

(c) $\frac{1}{9}$

(d) $-\frac{1}{9}$

7 The function $f : f(x) = 3 - \ln x^2$ increases in the interval ....

(a) $]-\infty , \infty[$

(b) $]-\infty , 0[$

(c) $]0 , \infty[$

(d) $]3 , \infty[$

8 $\lim\limits_{h \to 0} \frac{\cot(x + h) - \cot(x)}{h} =$ ....

(a) $-\csc^2 x$

(b) $\sec^2 x$

(c) $-\cot^2 x$

(d) $\cot x \csc x$

9 The curve of tangent slope at any point on it equals a $\csc^2 x$ where a is a constant , and the curve passes through the two points $\left(\frac{\pi}{4}, 5\right)$, $\left(\frac{3\pi}{4}, 1\right)$
Find the equation of the curve.

10 A circular sector whose perimeter is 30 cm. , and its area is as great as possible , find the radius length of its circle.

11 The curve $y = x^3 - 6x^2$ is convex downwards in the interval ....

(a) $\mathbb{R} -]0 , 4[$

(b) $]0 , 4[$

(c) $]2 , \infty[$

(d) $]-\infty , 2[$

12 ***Answer one of the following items :***

[a] Find : $\int x^3 e^{x^2} dx$

[b] Find : $\int \sec^{2017} x \tan x \, dx$

13 A 5 - metre water pipe with two ends A and B is leaning with its end A on a horizontal ground and with one of its points D against a 3 - metre vertical wall. If end A slides away from the wall at a rate $\frac{5}{4}$ metre/m. , find the rate of sliding end B when the pipe reaches the edge of the wall.

14 *Answer one of the following items :*

[a] **Find the area of the region bounded by the curves :**

$x = \sqrt{y}$, $x + y = 0$, $x - y + 6 = 0$

[b] **Find the area of the region bounded by the two curves :**

$y + x^2 = 6$, $y + 2x - 3 = 0$

15 If $y = e^{-X}\sqrt{\dfrac{1+X}{1-X}}$ where $-1 < X < 1$ **Prove that :** $(1 - X^2)\, \grave{y} = X^2 y$

16 The rate of change of $(X - \sin X)$ with respect to $(1 - \cos X)$ at $X = \dfrac{\pi}{3}$ equals ....

(a) $\dfrac{\sqrt{3}}{3}$

(b) $\sqrt{3}$

(c) $2\sqrt{3}$

(d) $\dfrac{2}{3}$

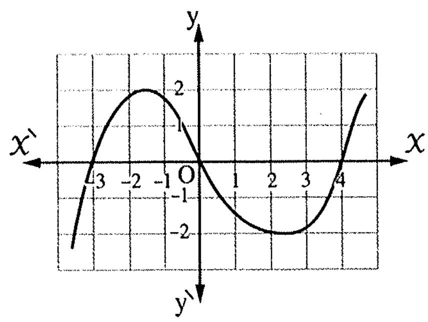
17 The opposite figure represents the curve $f''(X)$, the inflection is at $X =$ ....

(a) zero

(b) -3

(c) 4

(d) all the previous.



18 By using integration prove that the right circular cone equals $\frac{1}{3} \pi r^2 h$ where r is the radius length of its base , h is the height.

Model 16

Answer the following questions :

1 If the curve $y = (5x - a)^3 + 2$ has an inflection point at $x = 4$, then a = ...

(a) 20

(b) -20

(c) 5

(d) 4

2 The function $f : f(x) = -x^2$ is increasing in the interval ....

(a) $]-\infty , 0[$

(b) $]0 , \infty[$

(c) $]-\infty , \infty[$

(d) $]2 , \infty[$

3 The measure of the positive angle which the tangent to the curve : $y^2 + 2x^2 = 6$ at the point $(1 , 2)$ makes with the positive direction of x-axis = ...°

(a) 45

(b) 135

(c) 120

(d) 150

4 $\int_{-\pi}^{\pi} (4 + \pi \cos 2x) \, dx =$ ....

(a) π

(b) 2π

(c) 4π

(d) 8π

5 Find the equation of the tangent and normal to the curve : $x^2 - 2xy - y^2 = 1$ at $(1, 0)$

6 If $y = \frac{1}{x} \ln e^x$, then $\frac{dy}{dx} =$ ....

(a) 1

(b) e^x

(c) zero

(d) $\ln x$

7 ***Answer one of the following items :***

[a] Find : $\int \frac{dx}{\sqrt[3]{2x+9}}$

[b] Find : $\int \left[(1 - \cot x)^2 + 2 \cot x\right] dx$

8. Find the absolute maximum value and absolute minimum value of the function $f : f(X) = \sin X$ in the interval $\left[\frac{5\pi}{6}, \frac{11\pi}{6}\right]$

9. The area of the region bounded by the straight lines $2y = -X + 8$ and X-axis and the two straight lines $X = 2$, $X = 4$ equals .... square unit.

ⓐ 4
ⓑ 5
ⓒ 3
ⓓ 6

10. ABC is an equilateral triangle of side length 2ℓ , E is the midpoint of $\overline{BC}$, $D \in \overline{AB}$, $F \in \overline{AC}$ such that $\overline{DF} \parallel \overline{BC}$

Prove that : The greatest area of the triangle DEF $= \frac{1}{4}$ the area Δ ABC

11 A train starts its journey at 11 O'clock towards east with velocity 45 km./h. , while another train began its journey at 12 O'clock from the same point towards south with velocity 60 km./h. , find the rate of increasing of distance between the two trains at 3 O'clock afternoon.

12 $\lim\limits_{x \to 0} \left(1 + \frac{x}{a}\right)^{\frac{a}{x}} = \ldots\ldots\ldots\ldots\ldots$

(a) zero

(b) $\frac{1}{a}$

(c) $\frac{1}{e}$

(d) e

13 If the slope of the tangent to the curve of the function : $y = f(X)$ at any point (X, y) on it is $6X^2 - 30X + 36$ Find the equation of the curve given that it has local maximum value 28

14 A point moves on the curve : $y = x^2 - 4x + 1$, then the position of the point at which the rate of change of its x-coordinate is twice the rate of change of its y-coordinate with respect to time is ....

(a) (2.25 , 2.25)

(b) (5 , 2.25)

(c) $\left(\frac{9}{4} , \frac{-47}{16}\right)$

(d) $\left(\frac{9}{4} , \frac{9}{8}\right)$

15 If $y = e^{a x}$

Answer one of the following items :

[a] Prove that : $\frac{d^4 y}{d x^4} - a^4 y = 0$

[b] Find the value of a which verifies that : $\frac{d^2 y}{d x^2} - 2\frac{d y}{d x} + y = 0$

16 The given figure represents the curve $f''(x)$

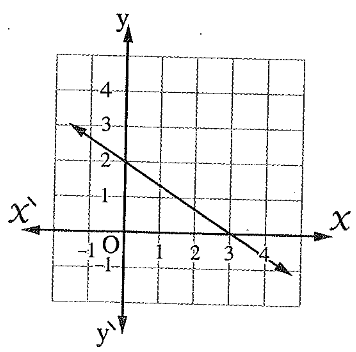
, then the curve of $f(x)$ is convex upwards at $x \in$ ....

(a) $]-\infty , 0[$

(b) $]-\infty , 3[$

(c) $]0 , \infty[$

(d) $]3 , \infty[$



17 The local minimum value of the function $f : f(X) = |X - 1|$ is at $X =$ ....

(a) zero

(b) 1

(c) 2

(d) 3

18 Find the volume of the solid generated by revolving the plane region bounded by the curve $y = -X^2 + 8$ and the straight lines $X = 2$, $X = 0$, $y = 4$ a complete revolution about y-axis.

Model 17

Answer the following questions :

1 $\lim\limits_{x \to 0} (1 + x)^{\frac{1}{3x}} = \dots\dots\dots\dots$

(a) $\frac{1}{3}$

(b) e^3

(c) $e^{\frac{1}{3}}$

(d) $\frac{e}{3}$

2 The inflection point of the curve $f : f(x) = (x - 1)(x^2 + x + 1)$ is $\dots\dots\dots\dots$

(a) $(1, 0)$

(b) $(0, 1)$

(c) $(-1, 0)$

(d) $(0, -1)$

3 The function $f : f(x) = -|x| + 1$ is decreasing in the interval $\dots\dots\dots\dots$

(a) $]-\infty, 1[$

(b) $]1, \infty[$

(c) $]0, \infty[$

(d) $]-\infty, 0[$

4 If $y = e^x$, $z = \sin x$, then $\frac{dy}{dz} = \dots\dots\dots\dots$

(a) $\frac{e^x}{\sin x}$

(b) $e^x \tan x$

(c) $e^x \cos x$

(d) $\frac{e^x}{\cos x}$

5 $\int_{-1}^{1} \frac{x^3}{x^4 + \cos x} \, dx = \dots\dots\dots\dots$

(a) -1

(b) zero

(c) 1

(d) 4

6 Find the equations of the two tangents to the circle $x^2 + y^2 = 5$ that each inclined to the positive x-axis by an angle whose tangent is 2

7 ***Answer one of the following items :***

[a] Find : $\int x^5 \left(1 + \frac{3}{x}\right)^5 dx$

[b] Find : $\int \frac{1 + \sin^2 x}{1 - \sin^2 x} \, dx$

8 If $y = \sin 3x + \cos 3x$ **Prove that :** $\frac{d^4 y}{d x^4} = 81 y$

9 A window is formed from a rectangle headed by a semi circle whose diameter is coincide with one of the sides of the rectangle , the perimeter of the window equals 6 metres , find the radius length of the semi circle such that the area of the window is to be maximum.

10 The rate of change of e^{x^3} with respect to $\ln x$ equals ....

(a) $3x^3 e^{x^3} + 3x^2$

(b) e^{x^3}

(c) $3x^3 e^{x^3}$

(d) $3x^2 e^{x^3}$

11 The curve $y = x^{\frac{1}{5}}$ at $(0 , 0)$ has ....

(a) a vertical tangent.

(b) a horizontal tangent.

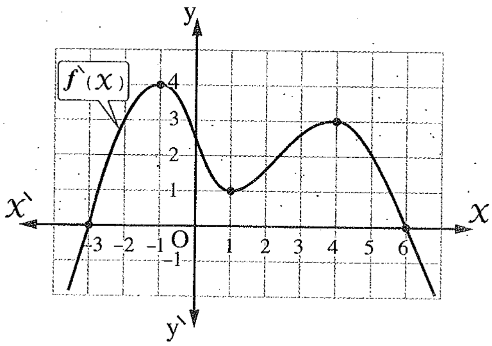
(c) an inclined tangent.

(d) no tangent.

12 If $y = f(x)$ represents a curve of polynomial function of third degree and $f''(x) < 0$ at $x < \frac{-2}{3}$, $f''(x) > 0$ at $x > \frac{-2}{3}$ and passes through the point $(1 , 6)$ and there is a critical point at $(-1 , 2)$ Find the equation of the curve and determine the kind of the critical point.

13 Find the area of the region bounded by the curve of the function $y = 4x^2 + 4x + 1$ and the two straight lines $x = 1$, $y = 1$

14 The opposite figure represents the curve $f'(x)$, then the curve has maximum value at $x =$ ....
and minimum value at $x =$ ....



(a) -3 , 6
(b) 6 , -3
(c) 6 , 0
(d) 0 , 3

15 ABC is a triangle , A (0 , 0) , B (5 , 0) , C (8 , 3) , then the volume of the solid generated by revolving this triangle a complete revolution about x-axis = ... cubic unit.

(a) 24π
(b) 18π
(c) 15π
(d) 9π

16 $\frac{d}{dx}\left[(\csc x - \cot x)(\csc x + \cot x)\right] = \dots\dots\dots$

(a) zero

(b) $\csc^2 x - \cot^2 x$

(c) $\csc x \cot x + \sec^2 x \tan x$

(d) $\csc x \cot x - \csc^2 x$

17 The slope of the tangent to a curve at any point (x, y) on it equals $\frac{\sqrt{2y+1}}{\sqrt{3x-2}}$, find the equation of the curve given that it passes through $(1, 4)$

Model examinations

18 A 170 cm. man ascends up an inclined plane with uniform velocity 6 m./min. , the plane makes angle whose tangent is $\frac{7}{24}$ and its length 25 m. , a lamp is fixed at height $11\frac{1}{4}$ m. above the horizontal plane passes by the base of the inclined plane and vertically above the top of the incline plane.

Answer one of the following items :

[a] Find the rate of decreasing of the man's shadow.

[b] Find the rate of approching of the end of the man's shadow to the top of the inclined plane.

Model 18

Answer the following questions :

1 The function $f : f(X) = X^3 - X^2$ is decreasing in the interval ....

(a) $]-\infty , \frac{2}{3}[$

(b) $]\frac{2}{3} , \infty[$

(c) $]0 , \frac{2}{3}[$

(d) $]0 , \infty[$

2 The curve $y = (2X - c)^3 + 4$ has an inflection point at $X = 5$, then c = ...

(a) 2

(b) 4

(c) 5

(d) 10

3 If $X = \sin y$, then $\frac{dy}{dX} =$ ....

(a) $\sqrt{1 - X^2}$

(b) $\frac{1}{\sqrt{1 - X^2}}$

(c) $\sqrt{X^2 - 1}$

(d) $\frac{1}{\sqrt{X^2 - 1}}$

4 If $X = z^2 - 2z$, $y = z^2$, then $\frac{d^2 y}{dX^2} =$ .... at z = 2

(a) $-\frac{1}{2}$

(b) $\frac{3}{4}$

(c) $\frac{9}{16}$

(d) $\frac{1}{48}$

5 ***Answer one of the following items :***

[a] If $x \in]0 , \pi[$ Find the local maximum and local minimum values of the function $f : f(x) = \sin x \cos x + 5$

[b] Investigate the convexity intervals of the function $f : f(x) = -3x^5 + 5x^3$, then find the inflection point (if existed)

6 $\int_0^1 x e^{x^2} \, dx = \ldots\ldots\ldots\ldots$

(a) $\frac{e+1}{2}$

(b) $\frac{e-1}{2}$

(c) $\frac{e}{2}$

(d) $\frac{1}{2}$

7 If $4y^3 = 3x^2$ **Prove that :** $2y^2 \frac{d^2y}{dx^2} + 4y\left(\frac{dy}{dx}\right)^2 = 1$

8 The circumference of a circle increases at a rate 10π cm./min.
, then the radius length increases at a rate ....

(a) $\frac{5}{2}$
(b) 5
(c) 10
(d) 20

9 The tangent to the curve $x = 3\cos\theta$, $y = 3\sin\theta$ where $0 \le \theta \le \pi$ is parallel to the x-axis if $\theta =$ ....

(a) zero
(b) $\frac{\pi}{3}$
(c) $\frac{\pi}{2}$
(d) π

10 The sides of right-angled triangle changes , and its perimeter remains constant 40 cm. the rate of change of the hypotenuse is 7 cm./min. when the side lengths 8 , 15 , 17 cm. Find the rate of change of each side of the right-angle at this moment.

11 If $y = \ln x$, then $\frac{d^{10} y}{d x^{10}} = \ldots\ldots\ldots\ldots$

ⓐ $\frac{\lfloor 9}{-x^{10}}$

ⓑ $\frac{\lfloor 10}{-x^{9}}$

ⓒ $\frac{\lfloor 9}{x^{10}}$

ⓓ $\frac{\lfloor 10}{x^{9}}$

12 Find the curve equation $y = f(x)$ given that $\frac{d^2 y}{d x^2} = \frac{2}{x^3}$ and the tangent equation to the curve at the point $\left(2, \frac{5}{2}\right)$ which lies on it is $3x - 4y + 4 = 0$

13 The maximum value of the expression $(\sin x + \cos x)$ is $\ldots\ldots\ldots\ldots$

ⓐ 1

ⓑ 2

ⓒ $\sqrt{2}$

ⓓ $\frac{1}{\sqrt{2}}$

14 ***Answer one of the following items :***

[a] Find : $\int \frac{x+2}{x^2+4x+3}\, dx$

[b] Find : $\int \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}\, dx$

15 $\lim_{t \to \infty} t\,[\ln(t+1) - \ln(t)] = \ldots\ldots\ldots\ldots$

(a) 1

(b) t

(c) e

(d) e^2

16 Find the area of the plane region bounded by the curve $y = 6 - x^2$ and the straight line passing through the two points $(3, -3)$, $(-2, 2)$

17 The volume of a solid generated by revolving the region bounded by the curve $y = x^3$ and the two straight lines $x = 0$ and $y = 1$ a complete revolution about x-axis is equal to the volume of cylinder-like wire whose length is 42 units.
What is the radius length of that wire ?

18 A wire of length 34 cm. is divided into two parts. A square is to be constructed from one part and a circle is to be constructed from the second part. Find the length of each part such that the sum of the area of the two figures is to be minimum.

Model 19

Answer the following questions :

1 $\lim\limits_{x \to 0} \frac{1-e^{2x}}{1-e^{x}} = \dots\dots\dots\dots$

(a) 1

(b) -1

(c) 2

(d) -2

2 $\int_{-1}^{1} \sqrt{1-|x|}\, dx = \dots\dots\dots\dots$

(a) $\frac{4}{3}$

(b) zero

(c) $\frac{2}{9}(2)^{\frac{3}{2}}$

(d) $\frac{-2}{9}(2)^{\frac{3}{2}}$

3 The tangent to the curve $x^2 - xy + y^2 = 27$ drawn from the point $(6, 3)$ makes an angle of measurement $\dots\dots\dots\dots$ with the positive x-axis.

(a) 90°

(b) 60°

(c) zero°

(d) 180°

4 If $x = \sec z$, $\sqrt{y} = \tan z$

, then : $\frac{d^2 y}{d x^2} = \dots\dots\dots\dots$

(a) $\tan z \sec z$

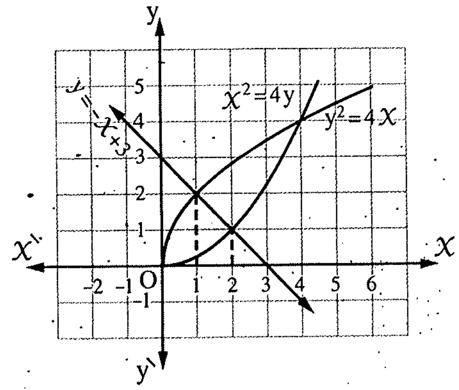
(b) $\sec^2 z \tan^2 z$

(c) 3

(d) 2

5 **In the given figure :**

Find the area of the region lies in the first quadrant and bounded by the curves $x + y = 3$, $x^2 = 4y$, $y^2 = 4x$



6 If the tangent to the curve $y = x^2$ passes through the point $(3, 5)$ Find the equation of this tangent.

7 If the curve of the function f lies above all tangents drawn from all points on the curve then the curve of the function is ....

(a) convex upwards.

(b) increasing.

(c) convex downwards.

(d) decreasing.

8 If the curve $y = x^3 + a x^2 + b x$ has an inflection point at $(3 , -9)$, then $a + 2b = \dots\dots\dots\dots$

ⓐ 15

ⓑ 21

ⓒ -27

ⓓ 6

9 If $y = \frac{1}{x}$, $x \neq 0$ **Prove that :** $x^2 \frac{d^2 y}{d x^2} + 3 x \frac{d y}{d x} + y = 0$

10 If f is a function where $f(x) = x\,|x - 2|$

Answer one of the following items :

[a] Determine the increasing and decreasing intervals of the function f

[b] Calculate the absolute maximum and absolute minimum values of the function in the interval $\left[\frac{1}{2} , \frac{5}{2}\right]$

11 If X is an acute angle , $f(X) = \ln(\sin X) - \ln(\cos X)$, then $f'\left(\frac{\pi}{4}\right) =$ ....

(a) 2

(b) -2

(c) 1

(d) -1

12 The slope of the tangent to a curve at any point on it equals $\sec^2 X - \sin X$, then the curve equation is .... given that it passes through $\left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right)$

(a) $y = \tan X + \cos X - \frac{1}{2}$

(b) $y = \tan X + \cos X + 1$

(c) $y = \tan X + \cos X$

(d) $y = \tan X + \cos X - 1$

13 An isosceles triangle , the length of each equal side is 10 cm. and the measure of their included angle $= X$, if the rate of change of X is 3 degree per minute , then the rate of change of area of the triangle at $X = 60°$ is ....

(a) 75 cm^2/min.

(b) $\frac{5}{12}\pi$ cm^2/min.

(c) $50\sqrt{3}$ cm^2/min.

(d) $\frac{5\sqrt{3}}{12}$ cm^2/min.

14 Find the volume of the solid generated by revolving the region bounded by the curve $\frac{X^2}{a^2} + \frac{y^2}{b^2} = 1$ and X-axis where a , b are constants a complete revolution about X-axis.

15 If y , z are two differentiable function in x , then $d\,(y^t z^m) =$ ....

(a) $t\,m\,y^{t-1}\,z^{m-1}$

(b) $y^t z^m \left(\frac{t}{y}\,d\,y + \frac{m}{z}\,d\,z\right)$

(c) $t\,y^{t-1}\,z^m + m\,y^t\,z^{m-1}$

(d) $t\,z^m\,d\,y^t + m\,y^t\,d\,z^m$

16 ***Answer one of the following items :***

[a] Find : $\int (\csc x + \sin x)^2\,d\,x$

[b] Find : $\int x \sec^2 x\,d\,x$

17 Find the points on the curve $x^2 - y^2 = 8$ where the distance between them and the point (0 , 2) is minimum.

18 A rope of length 25 m. passes over a pulley at height 12 m. from the ground , tied at one of its ends by a mass and the other end by a car moves with velocity 6 m./sec. Find the rate of change of height of the mass at the moment that the car at a distance 16 m. from the projection of the pulley on the ground.

Model 20

Answer the following questions :

1 $\int x \sqrt[5]{\frac{6}{x^4} + \frac{3}{x^5}}\, dx = \dots\dots\dots + C$

ⓐ $\frac{1}{5}(6x+3)^{-\frac{4}{5}}$

ⓑ $\frac{6}{5}(6x+3)^{\frac{6}{5}}$

ⓒ $\frac{5}{36}(6x+3)^{\frac{6}{5}}$

ⓓ $\frac{5}{36}(6x+3)^{-\frac{4}{5}}$

2 $\lim\limits_{x \to 0} \frac{e^{-x}-1}{3^x-1} = \dots\dots\dots$

ⓐ $-\ln 3$

ⓑ $\frac{1}{\ln 3}$

ⓒ $-\frac{1}{\ln 3}$

ⓓ $\ln 3$

3 The points which separate the upwards and downwards convexity intervals are called ....

ⓐ local maximum.

ⓑ local minimum.

ⓒ stationary.

ⓓ inflection.

4 The tangent equation to the curve $\sin x = \cos 3y$ at the point $\left(0, \frac{\pi}{6}\right)$ is ....

ⓐ $6y - 2x = \pi$

ⓑ $6y + 2x = \pi$

ⓒ $6y - 3x = \pi$

ⓓ $6y + 3x = \pi$

5 Find the area of the triangle bounded by the X-axis and the tangent and normal to the curve of the function $f : f(X) = X^2$ at the point $(2, 4)$ which lies on it.

6 If $y = x \tan x$ **Prove that :** $\frac{d^2 y}{d x^2} = 2 (1 + y) \sec^2 x$

7 ***Answer one of the following items :***

[a] Find : $\int (a^{3 \log_a x} + a^x + \cos \theta) \, d x$ where a and θ are constants.

[b] Find : $\int 8 \sin^2 \frac{x}{2} \cos^2 \frac{x}{2} \, d x$

8 The area of the region bounded by the curve $x y = 4$ and x-axis and the two straight lines $x = 1$, $x = 3$ equals ....

(a) 2 ln 3

(b) 4 ln 3

(c) 3 ln 3

(d) ln 3

9 $\frac{d}{dx}\left(2 \cot \frac{\pi}{4}\right) = \dots\dots\dots$

(a) $-2 \csc^2 \frac{\pi}{4}$

(b) $-2 \cot \frac{\pi}{4} \csc^2 \frac{\pi}{4}$

(c) 2

(d) zero

10 If the function $f : f(x) = x^2 + \frac{b}{x}$ has a critical point at $x = 2$, then b = $\dots\dots\dots$

(a) 16

(b) 8

(c) -16

(d) 4

11 A vessel full of liquid , the liquid leaks from a small hole in the bottom of the vessel , if the volume of the liquid changes at a rate $(0.4\,t - 40)$ cm^3/sec. , where t is the time in seconds and the liquid volume after 30 sec. from the beginning of leaking is 980 cm^3.
Find the capacity of the vessel and after how many seconds the vessel become empty.

12 The tangent to the curve $X = e^{\theta} \cos \theta$, $y = e^{\theta} \sin \theta$ at the point whose $\theta = \frac{\pi}{4}$ makes with the positive direction of X-axis an angle of measure ....

(a) zero

(b) $\frac{\pi}{4}$

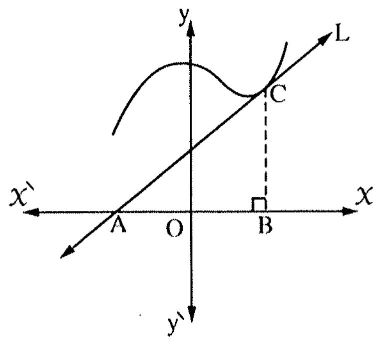
(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{2}$

13 **In the opposite figure :**

The straight line L is a tangent to the function f at the point C and intersects the X-axis at the point A $(-4 , 0)$, the coordinates of the point B $(4 , 0)$ and $f(4) + f'(4) = 9$

, then the area of Δ ABC = ... square unit.



(a) 36

(b) 72

(c) 32

(d) 18

14 One of the side of a rectangle lies on X-axis , and the vertices on the opposite side lie on the curve $y = 4 - X^2$ Find the dimensions of the rectangle which make the area as maximum as possible.

15 $\int_{-2}^{4} |x^2 - 3x| \, dx =$ ....

(a) 15

(b) 12

(c) – 15

(d) 60

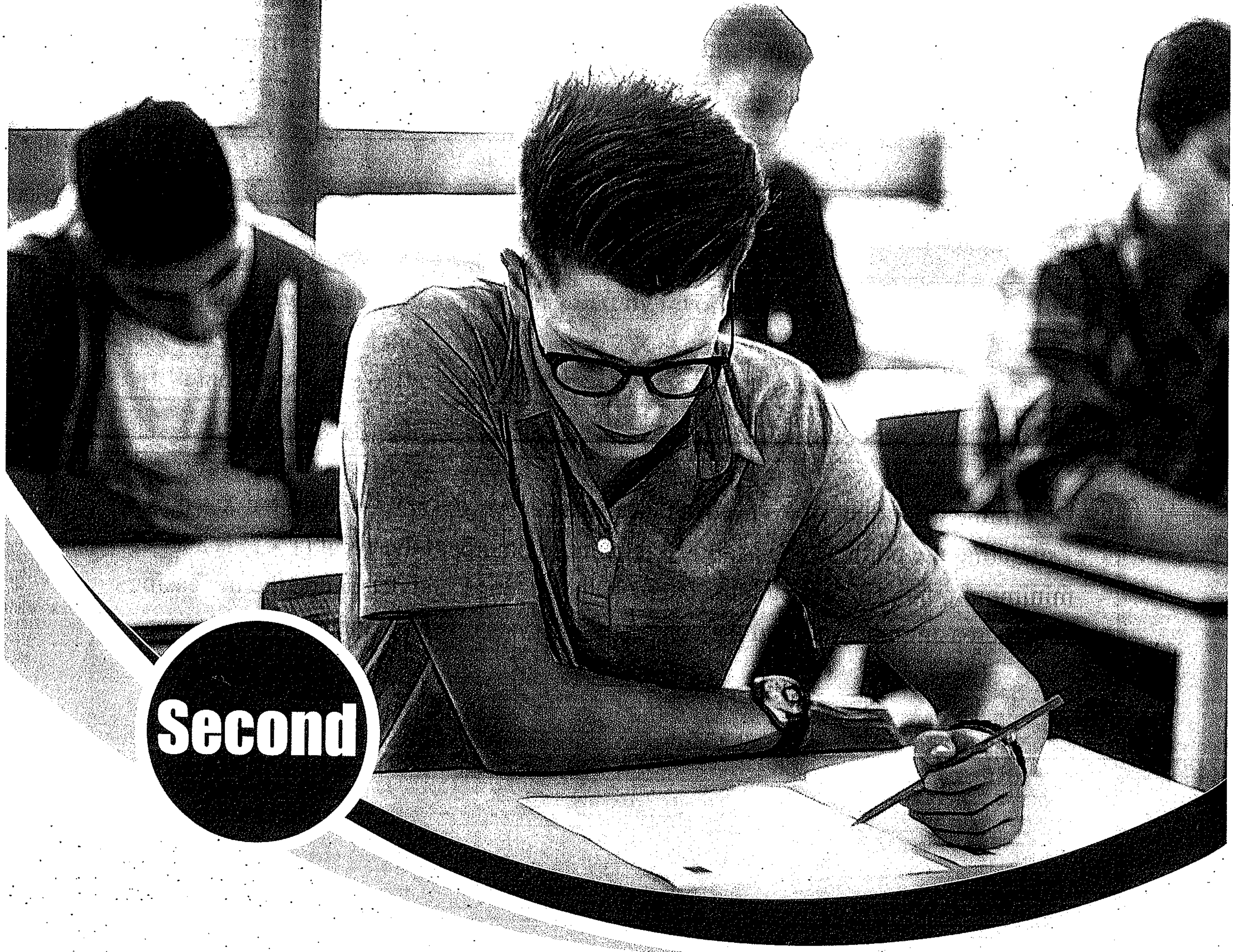
16 ***Answer one of the following items :***

[a] Find the volume of the solid generated by revolving the region bounded by the curve $y = x^2$ and the straight line $y = 2x$ a complete revolution about x-axis.

[b] Find the volume of the solid generated by revolving the region bounded by the curve $y = \frac{1}{x}$ and the straight line $x = 1$, $x = 4$, $y = 0$ a complete revolution about x-axis.

17 The length of two sides of the right angle in right-angled triangle are 8 , 6 cm. , the length of the first side decrease at a rate $\frac{1}{2}$ cm./min. and the length of the second side increase at a rate 1 cm./min. Find the rate of increasing in area of the triangle after two minutes , and the time at which the increasing vanished.

18 Determine the values of a , b , c , d where the curve $y = a x^3 + b x^2 + c x + d$, such that the point (0 , 8) is a local maximum point and the point (1 , 7) is a local minimum point , then sketch the curve of the function and determine the increasing and decreasing intervals and the convexity intervals upwards and downwards.



School Book Examinations

Model 1

First : Answer the following question :

1 **Choose the correct answer :**

(1) Which of the following functions satisfies the relation $\frac{d^3 y}{d x^3} = y$?

(a) $\frac{1}{12}(x+1)^4$ (b) $\sin x$ (c) e^x (d) $\frac{x}{x-1}$

(2) If the radius length of a circle increases at a rate $\frac{1}{\pi}$ cm./sec. the circumference of the circle increases at a rate of .... cm./sec.

(a) $\frac{2}{\pi}$ (b) 2 (c) π (d) 2π

(3) The curve of the function f where $f(x) = x^3 - 3x^2 + 2$ is convex upwards when $x \in$ ....

(a) $]-\infty , 0[$ (b) $]-\infty , 1[$ (c) $]1 , 3[$ (d) $]1 , \infty[$

(4) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (\sin x + \cos x)\, d x$ equals ....

(a) 4 (b) 2 (c) zero (d) π

(5) If f is a continuous function on $\mathbb{R}$, $\int_3^5 2 f(x)\, d x = 8$, $\int_3^4 3 f(x)\, d x = 9$, then $\int_4^5 5 f(x)\, d x$ equals ....

(a) zero (b) 1 (c) 3 (d) 5

(6) The area of the region bounded by the curve $y = \sqrt{16 - x^2}$ and x-axis measured in square units equals ....

(a) 16π (b) 12π (c) 8π (d) 4π

Second : Answer three questions only of the following :

2 **[a] Find :** $\int \sin x \cos^3 x\, d x$ « $-\frac{1}{4}\cos^4 x + c$ »

[b] If $e^{xy} - x^2 + y^3 = 0$, **find :** $\frac{d y}{d x}$ when $x = 0$ « $\frac{1}{3}$ »

---

3 **[a]** Find the equation of the tangent to the curve : $x^2 - 3xy - y^2 + 3 = 0$ at point $(-1 , 4)$

« $14x + 5y - 6 = 0$ »

[b] The lengths of the legs of the right angle of a right-angled triangle at a moment , are 6 cm. and 30 cm. If the length of the first leg increases at a rate of $\frac{1}{3}$ cm./min. and the length of the second leg decreases at a rate of 1 cm./min. , **find :**

(1) The rate of increase in the area of the triangle after 3 minutes.

(2) The time at which the increase of the area of the triangle stops. « 1 cm^2/min. , 6 »

4 [a] Determine the increasing and decreasing intervals to the function f where :
$f(x) = x + 2\sin x$, $0 < x < 2\pi$

[b] A rectangle is drawn such that two adjacent vertices of the rectangle lie on the curve $y = x^2 - 12$ and the other two vertices lie on the curve $y = 12 - x^2$, find the maximum area of this rectangle. « 64 »

---

5 [a] Find the volume of the solid generated by revolving the region bounded by the two curves $y = \frac{4}{x}$ and $y = (x-3)^2$ a complete revolution about x-axis « 5.4π cubic unit »

[b] Sketch the curve of the function f which satisfies the following properties :

(1) $f(1) = f(5) = 0$, $f(2) = -3$

(2) $f''(x) < 0$ for each $x \neq 2$

(3) $f'(x) < 0$ for each $x < 2$

(4) $f'(x) > 0$ for each $x > 2$

Model 2

First : Answer the following question :

1 **Choose the correct answer :**

(1) The equation of the tangent to the curve of the function f where $f(x) = e^{2x+1}$ at point $\left(\frac{-1}{2}, 1\right)$ is ....

(a) $2y = x + 1$ (b) $y = 2x + 2$ (c) $y = 2x - 3$ (d) $2y = 3x + 1$

(2) If $y = 4n^3 + 4$, $z = 3n^2 - 2$, then the rate of change of z with respect to y equals ....

(a) $2n$ (b) 2 (c) $\frac{1}{2n}$ (d) 4

(3) The maximum value of the expression : $8x - x^2$ where $x \in \mathbb{R}$ is ....

(a) 8 (b) 16 (c) 32 (d) 64

(4) If the slope of the tangent to the curve of the function f at any point on it equals $\frac{1}{x-2}$ and the curve passes through point $(3, 0)$, then $f(e^2 + 2)$ equals ....

(a) 2 (b) 3 (c) $\ln 2$ (d) $\ln 3$

(5) If f is a continuous function on $\mathbb{R}$, $\int_1^2 f(x)\,dx = 9$ and $\int_6^2 f(x)\,dx = -7$, then $\int_1^6 f(x)\,dx$ equals ....

(a) 2 (b) 8 (c) 16 (d) -63

(6) The volume of the solid generated by revolving the region bounded by the curve $y = \sqrt{x+1}$ and the straight lines $y = 0$, $x = -1$ and $x = 1$ equals ....

(a) π (b) $\frac{3\pi}{2}$ (c) 2π (d) $\frac{5\pi}{2}$

Second : **Answer three questions only of the following :**

2 **[a] Find :** (1) $\int x(2x-1)^3\, dx$ (2) $\int x e^{-2x}\, dx$

[b] Find the rate of change for : $\sqrt{16+x^2}$ with respect to $\frac{x}{x-2}$ when $x=-3$ « $\frac{15}{2}$ »

---

3 **[a]** If $x \cos y + y \cos x = 1$, **find :** $\frac{dy}{dx}$

[b] Find the absolute extrema values of the function f in the interval $[-1 , 1]$ where $f(x) = 2x^3 + 6x^2 + 5$ « 13 , 5 »

---

4 **[a]** If $f(x) = \begin{cases} 2x + x^2 & \text{when } x < 0 \\ 2x - x^2 & \text{when } x \geq 0 \end{cases}$

find : (1) The local maximum and minimum values of the function f

(2) $\int_{-1}^{3} f(x)\, dx$ « $-\frac{2}{3}$ »

[b] The volume of a cube increases regularly such that it keeps its shape at a rate of 27 cm^3/min. , find the increase of the area of its faces at the moment which its edge length is 3 cm. « 36 cm^2/min. »

---

5 **[a]** Find the area of the region bounded by the two curves : $y = x^2$ and $y = 6x - x^2$ (in square units). « 9 square unit »

[b] If the function f where $f(x) = x^3 + a x^2 + b x$ has an inflection point at (2 , 2) , find the two values of the two constants a and b , then sketch the curve of the function. « − 6 , 9 »

Model 3

First : **Answer the following question :**

1 **Choose the correct answer :**

(1) The slope of the tangent to the curve of the circle $x^2 + y^2 = 25$ when $x = 3$ equals ....

(a) $\frac{-4}{3}$ (b) $\frac{-3}{4}$ (c) $\frac{5}{12}$ (d) $\frac{4}{3}$

(2) If $f(x) = \frac{x}{x-2}$, then $f'''(3)$ equals ....

(a) − 36 (b) − 12 (c) 6 (d) 4

(3) If $\frac{dy}{dX} = \csc^2 X$, $y = 2$ and $X = \frac{\pi}{4}$, then y equals ....

(a) $-(2 + \cot X)$ (b) $-(3 + \cot X)$ (c) $2 - \cot X$ (d) $3 - \cot X$

(4) If $\int_2^4 f(X)\, dX = 7$, $\int_4^2 g(X)\, dX = 2$, then $\int_2^4 [2 f(X) - 3 g(X) - 5]\, dX$ equals ....

(a) -18 (b) -8 (c) 10 (d) 14

(5) The area of the region bounded by the straight lines :

$y = 2X - 3$, $y = X + 1$, $X = 2$ equals ....

(a) 2 (b) 3 (c) $\frac{9}{2}$ (d) 6

(6) The volume of the solid generated by revolving the region bounded by the two curves $y = \tan X$, and $y = \sec X$ and two straight lines $X = \frac{\pi}{6}$, $X = \frac{\pi}{3}$ a complete revolution about X-axis approximated in cubic units equals ....

(a) $\frac{\pi^2}{6}$ (b) $\frac{\pi^2}{3}$ (c) $\frac{2\pi^2}{5}$ (d) $2\pi^2$

Second : Answer three questions only of the following :

2 [a] Find the derivative of y with respect to X where $y = X^2 \ln X$ « $X(2 \ln X + 1)$ »

[b] If $f(X) = \sqrt[3]{(X-4)^2}$, find the convexity intervals upwards and downwards and the inflection points (if existed) to the curve of the function f

---

3 [a] Find : (1) $\int X (X - 5)^3\, dX$ (2) $\int 4 X e^{2X}\, dX$

[b] Find the absolute maximum values of the function f where :

$f(X) = X^4 - 4X^3$ on the interval $[0, 4]$ « 0 »

---

4 [a] The volume of a solid of revolution generated by revolving the region bounded by the curve $y = X^3$ and the two straight lines $X = 0$ and $y = 1$ a complete revolution about X-axis is equal to the volume of a cylinder-like wire whose length is 42 units. What is the radius length of that wire ? « $\frac{1}{7}$ length unit »

[b] The two equal legs of the isosceles triangle with a constant base whose length is ℓ cm. decrease at a rate of 3 cm./min. What is the rate of decrease in the area when the triangle becomes an equilateral triangle ? « $\sqrt{3}\, \ell$ cm²/min. »

---

5 [a] **Find the area of the region bounded by the two curves :** $X - y = 0$, $y = 4X - X^2$ « $\frac{9}{2}$ square unit »

[b] Sketch the curve of the continuous function f which has the following properties :

(1) $f(0) = 3$ (2) $f'(2) = f'(-2) = 0$

(3) $f'(X) > 0$ when $-2 < X < 2$

(4) $f''(X) < 0$ when $X > 0$, $f''(X) > 0$ when $X < 0$

Model 4

First : Answer the following question :

1 Choose the correct answer :

(1) If $y = \frac{3X - 5}{X - 2}$, then at $X = 1$, $\frac{d^3 y}{d X^3}$ equals ....

(a) -12 (b) -6 (c) 6 (d) 12

(2) $\int \sec^3 X \tan X \, d X$ equals ....

(a) $\frac{1}{4} \sec^4 X + c$ (b) $\frac{1}{3} \sec^3 X + c$ (c) $\frac{1}{2} \tan^2 X + c$ (d) $-\frac{1}{2} \tan^2 X + c$

(3) The normal to circle $X^2 + y^2 = 12$ at any point in it passes through point ....

(a) $(2 , 3)$ (b) $(1 , 1)$ (c) $(0 , 0)$ (d) $(-2 , -2)$

(4) The curve of the function f where $f(X) = (X - 2) e^X$ is convex downwards on the interval ....

(a) $]-\infty , \infty[$ (b) $]-1 , 2[$ (c) $]0 , 2[$ (d) $]0 , \infty[$

(5) $\int_{-1}^{3} 3X |X - 4| \, d X$ equals ....

(a) -27 (b) -20 (c) 20 (d) 27

(6) When the region bounded by the curve $X = \frac{1}{\sqrt{y}}$, $1 \le y \le 4$ and y-axis revolves a complete revolution about y-axis , then the volume of the solid generated measured in cubic units equals ....

(a) $\frac{2}{3} \pi$ (b) $3\sqrt{2} \pi$ (c) $2 \pi \ln 2$ (d) $\frac{2}{3} \pi \log 3$

Second : Answer three questions only of the following :

2 [a] Find : (1) $\int (3X^2 - 4 e^{2X}) \, d X$ (2) $\int \frac{X - 1}{\sqrt{X + 3}} \, d X$

[b] If $\sin y + \cos 2X = 0$ Prove that : $\frac{d^2 y}{d X^2} - \left(\frac{d y}{d X}\right)^2 \tan y = 4 \cos 2X \sec y$

3 [a] If $\int_1^4 f(X)\, dX = 7$, $\int_4^1 g(X)\, dX = 3$

Calculate the value of : $\int_1^4 [\grave{f}(X) + 2\, g(X) - 4]\, dX$ «– 11 »

[b] If the curve of the function f where $f(X) = aX^3 + bX^2 + cX + d$ has a local maximum value at (2 , 4) and an inflection point at (1 , 2) , find the equation of the curve.

« $f(X) = -X^3 + 3X^2$ »

---

4 [a] Find the area of the region bounded by the curve :

$\sqrt{X} + \sqrt{y} = 1$ and the two straight lines $X = 0$, $y = 0$ « $\frac{1}{6}$ square unit »

[b] Graph the curve of the continuous function f which satisfies the following properties :

(1) $f(4) = 2 f(3) = 4$ (2) $f(2) = 0$

(3) $\grave{f}(X) < 0$ when $X > 4$ or $X < 2$

(4) $\grave{\grave{f}}(X) < 0$ when $X > 3$, $\grave{\grave{f}}(X) > 0$ when $X < 3$

---

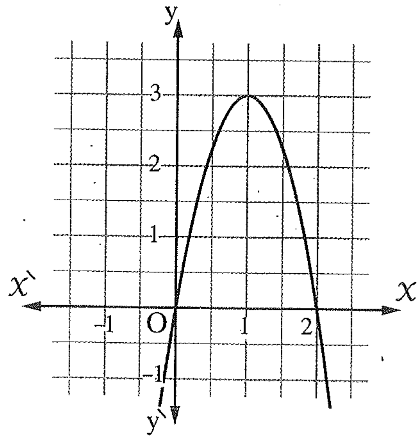
5 [a] Prove that the volume of the solid generated by revolving the region bounded by the two curves $y = \frac{4}{X}$ and $y = 5 - X$ just one revolution about X-axis equals 9π of the cubic units.

[b] If A is the area of the part bounded by two concentric circle whose radii lengths are r_1 and r_2 where $r_2 > r_1$, find the rate of change of A with respect to time at any moment at which $r_2 = 10$ cm. , $r_1 = 6$ cm. , if known that at this moment r_1 increases at a rate of 0.3 cm./s. and r_2 decreases at a rate of 0.2 cm./s. « -7.6π cm²/sec. »

Model 5

First : Answer the following question :

1 The opposite figure shows the curve $\grave{f}(X)$ of the function f where $f(X) = aX^3 + bX^2$, a , b are two constants.



Complete :

(1) The function f is decreasing for each $X \in$ ....

(2) The curve of f has critical points when $X \in$ ....

(3) The curve of f is convex upwards on the interval ....

(4) There is a local minimum value of the function f when $x =$ ....

(5) $f(1) =$ ....

(6) The area of the region bounded by the curve of the function f and the two straight lines $x = 2$, $x = 0$ and $y = 0$ in square units equals ....

Second : Answer three questions only of the following :

2 **[a] Find :**

(1) $\int \csc^2\left(\frac{x+5}{2}\right) dx$ (2) $\int \frac{5x}{3x^2-1} dx$

[b] The function f where $f(x) = x^3 - 6x^2 + 9x - 1$

(1) Determine the increasing and decreasing intervals of function f

(2) Find the maximum values of the function f in the interval $[0 , 2]$

---

3 **[a]** If $f(x) = 4 + \cot x - \sec^2 x$, find the equation of the normal to the curve of the function f at a point lying on the curve and its x-coordinate equals $\frac{\pi}{4}$

« $4x - 24y - \pi + 72 = 0$ »

[b] An empty tank whose capacity is 10 cubic metres. If the water is poured gradually in that tank at a rate of $(2t + 3)$ m^3/min. where t time in minutes , find the time needed to fill the tank. « 2 min. »

---

4 **[a] Find :** $\lim\limits_{x \to \infty} \left(\frac{2x-1}{2x+1}\right)^{2x}$ « $\frac{1}{e^2}$ »

[b] A rectangle - like poster contains 800 cm^2 of the printed material where the widths of both lower and upper margins are 10 cm. and the two side margins are 5 cm. what are the two dimensions of the posters which make its area as minimum as possible.

« 60 , 30 cm. »

---

5 **[a]** Find the volume of the solid generated by revolving the region bounded by the curve $y = 4 - x^2$ and the two positive parts of the axes of coordinates a complete revolution about x-axis. « $\frac{256}{15}\pi$ »

[b] If $f(x) = x^3 + a x^2 + b x + 4$ where a and b are two constants , find the two values of a and b if the function f has a local minimum value when $x = 2$ and an inflection point when $x = 1$, then sketch the curve of the function f

Model 6

First : Answer the following question :

1 **In each of the following phrases , choose (a) if the phrase is true and (b) if the phrase is false :**

(1) The local maximum value of the function is greater than its local minimum value. (a) (b)

(2) The rate of change of $\sqrt{n^2+3}$ with respect to $\frac{n}{n+1}$ is : $\frac{n\,(n+1)^2}{\sqrt{n^2+3}}$ (a) (b)

(3) If $\sqrt{y}-\sqrt{x}=2$, then $\frac{d^2 y}{d x^2}=\frac{-1}{x\sqrt{x}}$ (a) (b)

(4) $\int \frac{x-4}{(x-2)^6}\,d x=\frac{7\,(x-4)^2}{2\,(x-2)^7}+c$ (a) (b)

(5) If $y=x\ln x-x$, then $\frac{d y}{d x}=\ln x$ (a) (b)

(6) If $\left(a , f(a)\right)$ is an inflection point to the curve of the continuous function f , then $f''(a)=$ zero (a) (b)

Second : Answer three questions only of the following :

2 **[a] Find :**

(1) $\int \frac{7 x^3}{2-5 x^4}\,d x$ (2) $\int \left(3 e^{-5 x}+\frac{\pi}{x}\right) d x$

[b] If $y=a\,e^{x^2+1}$ **Prove that :** $\frac{d^3 y}{d x^3}=4\,x\,y\,(3+2\,x^2)$

---

3 **[a] Find :** $\int \cot x \csc^3 x\,d x$

[b] If s is the distance between point (1 , 0) and point (x , y) lying on the curve $y=\sqrt{x}$, find the coordinates of point (x , y) at which s is as minimum as possible. « $\left(\frac{1}{2} , \frac{1}{\sqrt{2}}\right)$ »

---

4 **[a]** Identify the absolute extrema values of the function f where $f(x)=|x|\,(x-4)$ in the interval $[-1 , 3]$

[b] If the slope of the tangent to the curve $y=f(x)$ at any point on it equals $6\,x^2+b\,x$ and $f(0)=5$, $f(2)=-3$, find the value of the constant b , then sketch the curve of the function f « $b=-12$ »

5 [a] Find the rate of change of $\ln(9 + x^3)$ with respect to $x^2 + 3$ and $x = 1$ «$\frac{3}{20}$»

[b] If A (0 , 3) , B (1 , 4) , C (2 , 0) **find using integration :**

(1) The surface area of ΔABC

(2) The volume of the solid generated by revolving Δ AOC a complete revolution about y-axis. «$\frac{5}{2}$ square unit , 4π cubic unit »

Model 7

First : Answer the following question :

1 **In each of the following phrases , choose (a) if the phrase is true and (b) if the phrase is false :**

(1) If $y^2 = 3x^2 - 7$, then $\frac{dy}{dx} = \frac{y}{3x}$ (a) (b)

(2) The function $f : f(x) = x^3 - 3x + 1$ has an inflection point which is (0 , 1) (a) (b)

(3) $\frac{d}{dx}[\cot(\cos 3x)] = 3\sin 3x \csc^2(\cos 3x)$ (a) (b)

(4) $\int (1 - \cos x)^4 \sin x \, dx = -\frac{1}{5}(1 + \cos x)^5 + c$ (a) (b)

(5) $\lim_{x \to \infty} \left(1 + \frac{5}{x}\right)^x = e^5$ (a) (b)

(6) $\int \left(\frac{2e}{x} + \frac{x}{e}\right) dx = 2e \ln|x| - \frac{x^2}{e} + c$ (a) (b)

Second : Answer three questions only of the following :

2 [a] **Find :**

(1) $\int x \sin x \, dx$ (2) $\int_{-1}^{1} \sqrt{x^4 + x^2} \, dx$

[b] Find the equation of the tangent to the curve $y = \ln\left(2 - \sqrt{2}\cos x\right)$ at the point lying on it and its x-coordinate equals $\frac{\pi}{4}$ «$x - y - \frac{\pi}{4} = 0$»

---

3 [a] Identify the convexity intervals downwards and upwards and the inflection points (if existed) to the curve of the function f where $f(x) = (x - 1)^4 + 3$

[b] A cuboid of metal whose base is square. If the side length of the base increases at a rate of 0.4 cm./sec. and the height decreases at a rate of 0.5 cm./sec. , find the rate of change of the volume when the side length of the base is 6 cm. and the height is 5 cm.

« 6 cm^3/sec. »

---

4 [a] **Find :** $\int_0^3 X\sqrt{X+1}\, dX$ « $\frac{116}{15}$ »

[b] A rectangle - like playground in which two opposite sides end in a semi-circle outside the rectangle of a diameter length equal to the length of this side.

If the perimeter of the playground is 400 metres , prove that the surface area of the playground is as maximum as possible when the ground is a circle - like , then find its radius length.

« $\frac{200}{\pi}$ »

---

5 [a] If $f(X) = X^3 - 3X + 3$, **find :**

(1) The absolute extrema value of the function f in the interval f $[0 , 2]$

(2) The area of the region bounded by the curve of the function f and the straight lines $X = 0$, $X = 2$, $y = 0$ « 4 square unit »

[b] Find the volume of the solid generated by revolving the region bounded by the curve $Xy = 2$ and the two straight lines $X = 1$ and $X = 2$ about X-axis « 2 π cubic unit »

Model 8

First : Answer the following question :

1 **Complete the following :**

(1) If $X^3 y^2 = 1$, then $\left[\frac{dy}{dX}\right]_{y=1} =$ ....

(2) $\frac{d}{dX}[7\, e^{\sec X}] =$ ....

(3) The function $f : f(X) = X^3 - 3X - 1$ has an inflection point which is ....

(4) If f is a continuous function on the interval $[2 , 7]$

, then $\int_2^7 f(X)\, dX + \int_7^4 f(X)\, dX =$ ....

(5) The area of the region bounded by the two curves $y = X^2$ and $y = 4X$ equals .... square units.

(6) If $y = X^2 \ln \frac{X}{a}$, $a \neq 0$, then $\left[\frac{d^3 y}{dX^3}\right]_{X=4} =$ ....

Second : Answer three questions only of the following :

2 **[a] Find :**

(1) $\int \frac{(x+3)^3 - 27}{x} \, dx$ (2) $\int x^2 e^{-x} \, dx$

[b] Find the equation of the tangent to the curve of the function f where $f(x) = 2 \tan^3 x$ at the point lying on the curve of the function f and its x - coordinate equals $\frac{\pi}{4}$

« $y = 12x - 3\pi + 2$ »

---

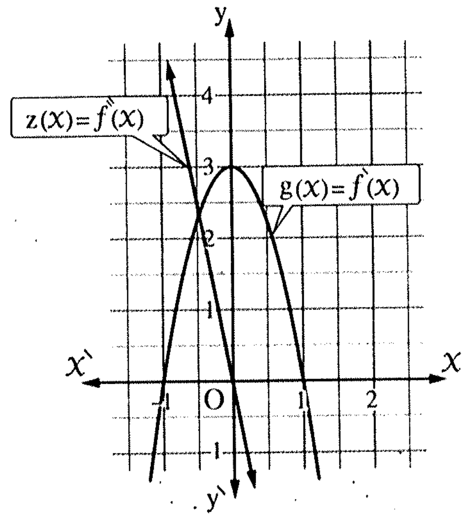
3 **[a] Find :** $\int_0^5 |x - 2| \, dx$ « $\frac{13}{2}$ »

[b] The opposite figure shows the two curves of the two functions g and z where :

$g(x) = f'(x)$, $z(x) = f''(x)$ and f is a polynomial function at the variable x

Sketch the curve of f knowing that it passes through the two points

$(-1, 0)$, $(1, 4)$



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4 **[a]** Identify the absolute extrema values of the function f in the interval $[0, 2]$, where $f(x) = 3\sqrt{4 - x^2}$

[b] A five metre length rod is fixed by a hinge to the ground at its base. If its top rises up by a winch at a rate of 1 m./min. , find the rate of decreasing the projector length of the rod on the ground when the height of the top is 3 metres. « $\frac{3}{4}$ m./min. »

---

5 **[a]** If a trapezoid is drawn in a semi-circle such that its base its the diameter of the semi-circle , determine the measure of the angle of the trapezoid base such that its area is as maximum as possible. « 60° »

[b] If a is the region bounded by the curve $x y = 4 + x^2$ and the straight lines $x = 1$, $x = 4$ and $y = 0$, **find :**

(1) The area of region a in square units to the nearest unit. « $4 \ln 4 + \frac{15}{2}$ square unit »

(2) The volume of the solid generated by revolving the region about x-axis.

« 57π cubic unit »

Model 9

First : Answer the following question :

1 Choose the correct answer :

(1) If $x = 2n^2 + 7$, $y = \sqrt{n^3}$, $n = 1$, then $\frac{dy}{dx}$ equals ....

(a) $\frac{3}{8}$ (b) $\frac{3}{4}$ (c) 2 (d) 6

(2) The curve of the function f is convex downwards on $\mathbb{R}$ if : $f(x)$ equals ....

(a) $2 - x^2$ (b) $2 + x^3$ (c) $2 - x^4$ (d) $2 + x^4$

(3) If the curve of the function $f : f(x) = x^3 + k x^2 + 4$, $K \in \mathbb{R}$ has an inflection point when $x = 2$, then k = ...

(a) – 6 (b) – 3 (c) 6 (d) 9

(4) If f is a continuous function on $\mathbb{R}$, $\int_{-1}^{3} f(x)\,dx = 7$, $\int_{5}^{3} f(x)\,dx = -11$, then $\int_{-1}^{5} f(x)\,dx$ equals ....

(a) – 4 (b) 18 (c) – 18 (d) 77

(5) $\int_{-1}^{3} |x - 1|\,dx$ equals ....

(a) – 6 (b) 0 (c) 4 (d) 8

(6) The area of the region bounded by the curve $y = x^3$ and the two straight lines $y = 0$ and $x = 2$ equals ....

(a) 1 (b) 2 (c) 4 (d) 8

Second : Answer three questions only of the following :

2 [a] Find : (1) $\int \frac{3x}{x^2 - 1}\,dx$ (2) $\int 9x^2 e^{3x}\,dx$

[b] Find the measure of the positive angle which the tangent of the curve $y^3 = x^2$ makes with the positive direction of x-axis when $x = 8$ to the nearest minute. « 18° 26 »

3 [a] If $\sin x = x y$, prove that : $x^2 (y + \ddot{y}) + 2\cos x = 2y$

[b] If the curve $y = 2x^3 + 3x^2 + 4x + 5$ has two parallel tangent , one of them touches the curve at point (– 1 , 2) , find the equation of the other tangent. «$4x - y + 5 = 0$»

4 [a] A balloon rises up vertically at a constant rate of 28 m./min. If the balloon is observed by a ground observer distant 200 m. away from the site of launching the balloon , find the rate of change of the angle of elevation of the observer when the balloon is 200 m. up. « 0.07 rad./min. »

[b] If the slope of the tangent to the curve of the function f at any point (X , y) on the curve is $3(X^2 - 1)$, find the local maximum and minimum values to the curve of the function f and the inflection points if existed known that the curve passes through the point $(-2 , -1)$, then sketch this curve.

---

5 The straight line $\overleftrightarrow{AB}$ intersects the curve of the function f at point $C(X , y)$, where $X > 0$, $A(0 , 2)$, $B(6 , 4)$ and $f(X) = \frac{9}{X}$, **find :**

(1) The equation of the straight line $\overleftrightarrow{AB}$ « $y = \frac{1}{3}X + 2$ »

(2) The coordinates of point C « (3 , 3) »

(3) The equation of the normal on the curve of f at point C and prove that it passes through the origin point O « $X - y = 0$ »

(4) The volume of the solid generated by revolving the region bounded by the normal $\overleftrightarrow{OC}$ and the curve of the function and the straight line $X = 6$ and X-axis a complete revolution about X-axis. « $\frac{45}{2}\pi$ cubic unit »

Model 10

First : Answer the following question :

1 **Complete :**

(1) $\lim\limits_{X \to \infty} \left(1 + \frac{1}{X}\right)^{X+3} =$ ....

(2) $\frac{d}{dX}(5 - 2\cot X)^3 =$ ....

(3) If the function $f : f(X) = kX^3 + 9X^2$ has an inflection point when $X = -1$, then k = ...

(4) $\int_{-1}^{3} (4X^3 - 6X^2 + 5)\, dX =$ ....

(5) If f is a continuous function on the interval $[1 , 4]$, then $\int_{1}^{4} f(X)\, dX + \int_{4}^{1} f(X)\, dX =$ ....

(6) The area of the region bounded by the two curves $y = X^4 + 1$ and $y = 2X^2$ equals .... square units.

Second : Answer three questions only of the following :

2 [a] **Find :** (1) $\int \tan (3X+1)\, dX$ (2) $\int (1-X^2)(3X-X^3)^5\, dX$

[b] If the two parametric equations of the function f where $y = f(X)$ are :

$X = 2n^3 + 3$ and $y = n^4$, find each of the following when $n = 1$

(1) The equation of the tangent to the curve of the function f

(2) $\dfrac{d^2y}{dX^2}$ « $2X - 3y - 7 = 0$, $\frac{1}{9}$ »

---

3 [a] Investigate the convexity of the curve of the function f where $f(X) = |X^3 - 1|$ and show the inflection points if existed.

[b] If $\int_{-2}^{3} f(X)\, dX = 9$, $\int_{5}^{3} f(X)\, dX = 4$

, **find the value of :** $\int_{-2}^{5} [3f(X) - 6X]\, dX$ « -48 »

---

4 [a] **Find the area of the plane region bounded by the two curves :**

$y + X^2 = 6$, $y + 2X - 3 = 0$ « $\frac{32}{3}$ square unit »

[b] A right circular cylinder-like container of internal height 9 cm. and the interior radius length of its base is 6 cm. A metal rod of length 16 cm. is placed in the container. If the rate of sliding the rod away from the edge of the cylinder is 2 cm./sec. , find the rate of sliding the rod on the cylinder base when the rod reaches the end of its base.

« $\frac{5}{2}$ cm./sec. »

---

5 [a] If the rate of change of the slope of the tangent to a curve at any point (X , y) on it is $6(1 - 2X)$ and the curve has a critical point when $X = 1$ and the function has a local minimum value equals 4

(1) Find the equation of the normal to the curve when $X = -1$

(2) Sketch the curve of the function and show the maximum and minimum values and the inflection points if existed. « $X - 12y + 109 = 0$ »

[b] Find the volume of the solid generated by revolving the plane region bounded by the curves : $y = X^3 + 1$, $y = 0$ and $X = 0$, $X = 1$ a complete revolution about X-axis.

« $\frac{23}{14}\pi$ cubic unit »

Guide Answers of

Egypt Exams and Model Examinations

Answers of Egypt exams

Answers of first session 2017

1 (a)

2 (b)

3 (d)

4

[a] $f(x) = (2-x)\,e^x$

$, f'(x) = (2-x)\,e^x + e^x(-1) = e^x(1-x)$

$, f''(x) = e^x(-1) + (1-x)\,e^x = -x\,e^x$

put $f'(x) = 0 \qquad \therefore e^x(1-x) = 0$

$\therefore 1 - x = 0 \qquad \therefore x = 1$

$, \because f''(1) = -e$ (negative)

$\therefore$ There exist a local maximum value at $x = 1$ and its value $= f(1) = e$

[b] $\because f(x) = 3x^4 - 4x^3$

$\therefore f'(x) = 12x^3 - 12x^2$

put $f'(x) = 0 \qquad \therefore 12x^3 - 12x^2 = 0$

$\therefore 12x^2(x-1) = 0$

$\therefore x = 0 \in [-1, 2]\ , x = 1 \in [-1, 2]$

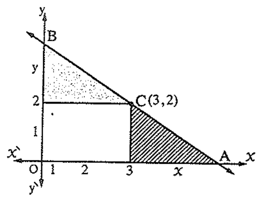
$, \because f(0) = $ zero $, f(1) = -1$

$, f(-1) = 7 \ , f(2) = 16$

$\therefore$ The absolute maximum value $= 16$ and the absolute minimum value $= -1$

5 (a)

6



From similarity of the triangles

$\frac{x}{3} = \frac{2}{y} \qquad \therefore y = \frac{6}{x}$

The area of ΔAOB :

$A = \frac{1}{2}(x+3)(y+2)$

$= \frac{1}{2}(x+3)\left(\frac{6}{x}+2\right) = 6 + x + 9x^{-1}$

$\therefore A' = 1 - 9x^{-2} \qquad \therefore A'' = 18x^{-3}$

Putting $A' = 0 \qquad \therefore 1 - \frac{9}{x^2} = 0$

$\therefore x^2 = 9 \ ,$ then $x = 3$

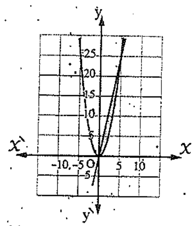
$A''_{x=3} > 0$ (i.e. minimum value)

$\therefore$ The smallest area : $A_{x=3} = 6 + 3 + \frac{9}{3}$

$= 12$ square unit.

7 (a)

8



To find the intersection points

we put $x^2 = 5x$

$\therefore x^2 - 5x = 0$

$\therefore x(x-5) = 0$

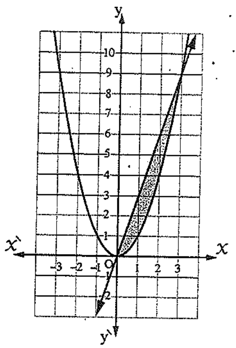
$\therefore x = 0$ or $x = 5$

$\therefore$ The area $= \int_0^5 (5x - x^2)\,dx$

$= \left[\frac{5x^2}{2} - \frac{1}{3}x^3\right]_0^5$

$= \frac{125}{2} - \frac{125}{3} = \frac{125}{6}$ area unit

9



To find the intersection points

we put $x^2 = 3x \qquad \therefore x^2 - 3x = 0$

$\therefore x(x-3) = 0$

$\therefore x = 0$ or $x = 3$

$\therefore$ The volume $= \pi \int_0^3 \left[(3x)^2 - (x^2)^2\right] dx$

$= \pi \int_0^3 (9x^2 - x^4)\,dx$

$= \pi \left[3x^3 - \frac{1}{5}x^5\right]_0^3$

$= \pi\left(81 - \frac{243}{5}\right) = \frac{162}{5}\pi$ volume unit.

10

[a] $\int \frac{x}{x+1}\,dx = \int \frac{x+1-1}{x+1}\,dx = \int\left(1 - \frac{1}{x+1}\right)dx$

$= x - \ln|x+1| + c$

[b] $\int x^2 \ln x\,dx$

x^2	$\ln x$
$\frac{1}{3}x^3$	$\frac{1}{x}$

$= \frac{1}{3}x^3 \ln x$

$- \frac{1}{3}\int x^2\,dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$

11 (d)

12 (c)

13 (a)

14

$\because y = 3e^x \qquad \therefore y' = 3e^x$

at $x = -1 \qquad \therefore y = 3e^{-1} = \frac{3}{e}$

$, y' = 3e^{-1} = \frac{3}{e}$

$\therefore$ The slope of the tangent $= \frac{3}{e}$

and the slope of the normal $= \frac{-e}{3}$

$\therefore$ The equation of the normal is

$\left(y - \frac{3}{e}\right) = \frac{-e}{3}(x+1)$

i.e. $y = \frac{-e}{3}x - \frac{e}{3} + \frac{3}{e}$

15 (a)

16 (c)

17

$\because x \times y = \frac{z+1}{z-1} \times \frac{z-1}{z+1} = 1. \qquad \therefore y = \frac{1}{x} = x^{-1}$

$\therefore y' = -x^{-2} \qquad \therefore y'' = 2x^{-3}$

$,$ at $z = 0 \qquad \therefore x = -1$

$\therefore y'' = 2(-1)^{-3} = -2$

18

$\because A = \pi r^2 \qquad \therefore \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

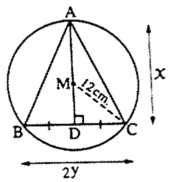
after 5 seconds $\qquad \therefore r = 4 \times 5 = 20$ cm.

$\therefore \frac{dA}{dt} = 2\pi \times 20 \times 4 = 160\pi$ cm^2/sec.

Answers of second session 2017

1 (b)

2



Let the triangle is ABC

, BC = 2 y cm.

, AD (hight of Δ) = X cm.

where each of X , 2 y ∈]0 , 24[

$\therefore A = \frac{1}{2}(2y) . X = Xy$

from properties of Δ MDC :

$y = \sqrt{12^2 - (X-12)^2} = \sqrt{24X - X^2}$

$\therefore A = X\sqrt{24X - X^2}$

$\acute{A} = \sqrt{24X - X^2} + X \times \frac{24 - 2X}{2\sqrt{24X - X^2}}$

$= \frac{24X - X^2 + 12X - X^2}{\sqrt{24X - X^2}} = \frac{36X - 2X^2}{\sqrt{24X - X^2}}$

let $\acute{A} = 0$ $\quad\therefore X = 0$ refused

or $X = 18$

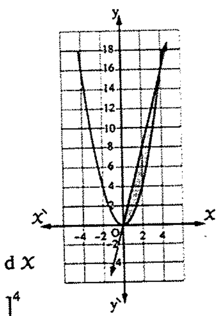
$\because$ The sign of $\acute{A}$ changed about two sides $X = 18$

$\therefore$ The area of triangle is Max.

$[A]_{X=18} = 108\sqrt{3}$ cm^2.

3 (c)

4



by solving the two equation

we get $X^2 = 4X$

$\therefore X^2 - 4X = 0$

$\therefore X(X - 4) = 0$

, then $X = 0$

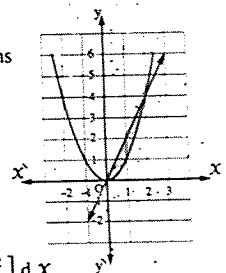
, $X = 4$

$\therefore$ the area $= \int_0^4 (4X - X^2)\,dX$

$= [2X^2 - \frac{1}{3}X^3]_0^4$

$= \frac{32}{3}$ square units

5



by solving the two equations

$X^2 = 2X$

$\therefore X^2 - 2X = 0$

$\therefore X(X - 2) = 0$

, then $X = 0$, $X = 2$

$\therefore$ the volume

$= \pi \int_0^2 [(2X)^2 - (X^2)^2]\,dX$

$= \pi \int_0^2 (4X^2 - X^4)\,dX$

$= \pi [\frac{4}{3}X^3 - \frac{1}{5}X^5]_0^2 = \pi [\frac{32}{3} - \frac{32}{5}]$

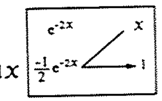
$= \frac{64}{15}\pi$ cubic unit

6

[a] $\int \frac{X}{3X^2+1}\,dX = \frac{1}{6}\int \frac{6X}{3X^2+1}\,dX$

$= \frac{1}{6}\ln|3X^2+1| + c$

[b] $\int \frac{X}{e^{2X}}\,dX = \int X . e^{-2X}\,dX$



$= \frac{-1}{2}Xe^{-2X} - \int \frac{-1}{2}e^{-2X}\,dX$

$= \frac{-1}{2}Xe^{-2X} + \frac{1}{2}\left(\frac{-1}{2}e^{-2X}\right) + c$

$= \frac{-1}{2}Xe^{-2X} - \frac{1}{4}e^{-2X} + c$

7 (d)

8 (a)

9

$y = X \sin X$ (Differentiate with respect to X)

$\therefore \frac{dy}{dX} = X\cos X + \sin X$

(Differentiate with respect to X)

$\therefore \frac{d^2y}{dX^2} = -X\sin X + \cos X + \cos X$

$= -y + 2\cos X$ (Differentiate with respect to X)

$\therefore \frac{d^3y}{dX^3} = -\frac{dy}{dX} - 2\sin X$ (by Multiplying × X)

$\therefore X\frac{d^3y}{dX^3} = -X\frac{dy}{dX} - 2X\sin X$

$\therefore X\frac{d^3y}{dX^3} + X\frac{dy}{dX} + 2y = 0$

10

let the length and width of the rectangle

after tine t sec are.

L = 24 − 2 t , w = 10 + 1.5 t

$\therefore$ the area (A) = (24 − 2t) (10 + 1.5 t)

$\therefore A = 240 + 16t - 3t^2$

$\therefore \frac{dA}{dt} = 16 - 6t$

After 4 sec. : $\frac{dA}{dt} = 16 - 6 \times 4 = -8$ cm^2/sec.

the area stoped increasing when $\frac{dA}{dt} = 0$

$\therefore t = \frac{16}{6} = \frac{8}{3}$ sec.

11 (b)

12 (d)

13 (c)

14

$\because y = (X^3 + 5)^X$ (by taking logarithm of both sides)

$\therefore \ln y = X\ln(X^3 + 5)$

$\frac{1}{y}\frac{dy}{dX} = \ln(X^3 + 5) + X \times \frac{3X^2}{X^3 + 5}$

$\therefore \frac{dy}{dX} = (X^3 + 5)^X \left[\ln(X^3 + 5) + \frac{3X^3}{X^3 + 5}\right]$

15 (b)

16 (b)

17 (a)

18

[a] $f(X) = X^3 - 3X^2 - 9X$

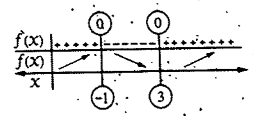
$\therefore f'(X) = 3X^2 - 6X - 9$

let $f'(X)$ = zero

$\therefore 3X^2 - 6X - 9 = 0$

$\therefore 3(X - 3)(X + 1) = 0$

$\therefore X = 3$, $X = -1$



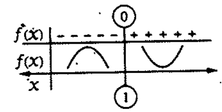
$\therefore$ local minimum value $f(3) = -27$

local maxinium value $f(-1) = 5$

$\because f''(X) = 6X - 6$

let $f''(X) = 0$

$\therefore X = 1$



$\therefore$ The inflection point (1 , − 11)

[b] $f(X) = 10Xe^{-X}$

$\therefore f'(X) = -10Xe^{-X} + 10e^{-X} = 10e^{-X}(1 - X)$

let $f'(X) = 0$

$\therefore 10e^{-X}(1 - X) = 0$

$\therefore 1 - X = 0$ $\quad\therefore X = 1$

X	zero	1	4
$f(X)$	zero	$\frac{10}{e}$	$\frac{40}{e4}$

- Absolute Maximum value $= \frac{10}{e} \approx 3.68$
- Absolute Minimum value = zero.

Answers of first session 2018

1 (b) **2** (c)

3

[a] $\int x^3 (x^2+1)^6 \, dx$

Put $z = x^2 + 1 \quad \therefore dz = 2x\,dx$

$\therefore x\,dx = \frac{1}{2} dz$

$\therefore \int x^3 (x^2+1)^6 \, dx = \int x^2 \cdot x (x^2+1)^6 \, dx$

$= \int (z-1) z^6 \times \frac{1}{2} dz$

$= \int \left(\frac{1}{2} z^7 - \frac{1}{2} z^6\right) dz$

$= \frac{1}{16} z^8 - \frac{1}{14} z^7 + c$

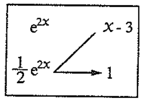
$= \frac{1}{16} (x^2+1)^8$

$- \frac{1}{14} (x^2+1)^7 + c$

[b] $\int (x-3) e^{2x} \, dx$

$= \frac{1}{2} (x-3) \times e^{2x} - \int \frac{1}{2} e^{2x} \, dx$

$= \frac{1}{2} (x-3) e^{2x} - \frac{1}{4} e^{2x} + c$



4 (a) **5** (b)

6

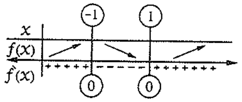
[a] $f(x) = x^3 - 3x - 2$

$\therefore f'(x) = 3x^2 - 3$

$\therefore f''(x) = 6x$

Put $f'(x) = 0 \quad \therefore 3x^2 - 3 = 0$

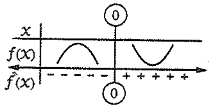
$\therefore x = 1$ or -1



$\therefore$ The function has local maximum value at $x = -1$, $f(-1) =$ zero

, the function has local minimum value at $x = 1$, $f(1) = -4$

Put $f''(x) = 0 \quad \therefore 6x = 0 , x = 0$



, $f(0) = -2$

there exist an inflection point at $(0 , -2)$

[b] $f(x) = x(x^2 - 12) = x^3 - 12x$

$\therefore f'(x) = 3x^2 - 12$

let $f'(x) = 0 \quad \therefore x = 2 \in [-1 , 4]$

or $x = -2 \notin [-1 , 4]$

$\because f(-1) = 11 , f(2) = -16 , f(4) = 16$

then the function has absolute maximum value $= 16$ at $x = 4$, and has absolute minimum value $= -16$ at $x = 2$

7 (a) **8** (b)

9

$x = \sec\theta \qquad \therefore \frac{dx}{d\theta} = \sec\theta \tan\theta$

, $y = \tan\theta \qquad \therefore \frac{dy}{d\theta} = \sec^2\theta$

$\therefore \frac{dy}{dx} = \frac{\sec^2\theta}{\sec\theta \tan\theta} = \csc\theta$

$\therefore \left(\frac{dy}{dx}\right)_{\theta = \frac{\pi}{6}} = 2 \qquad \therefore$ Slope of the tangent $= 2$

, slope of the normal $= \frac{-1}{2}$

at $\theta = \frac{\pi}{6}$ then : $x = \frac{2\sqrt{3}}{3}$, $y = \frac{\sqrt{3}}{3}$

$\therefore$ The point $\left(\frac{2\sqrt{3}}{3} , \frac{\sqrt{3}}{3}\right)$

Equation of the tangent : $\frac{y - \frac{\sqrt{3}}{3}}{x - \frac{2\sqrt{3}}{3}} = 2$

i.e. $2x - y - \sqrt{3} = 0$

$\therefore$ Equation of the normal : $\frac{y - \frac{\sqrt{3}}{3}}{x - \frac{2\sqrt{3}}{3}} = \frac{-1}{2}$

i.e. $3x + 6y - 4\sqrt{3} = 0$

10

$\sin y + \cos 2x = 0$ (by differentiate with respect to x)

$\therefore \cos y \frac{dy}{dx} - 2 \sin 2x = 0$ (by differentiate with respect to x)

$\therefore \cos y \frac{d^2y}{dx^2} + \frac{dy}{dx} \times - \sin y \times \frac{dy}{dx} - 4 \cos 2x = 0$

(divide by cos y)

$\therefore \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 \tan y = 4 \cos 2x \sec y$

11 (b) **12** (b)

13

$y = a x^b$ (by differentiate with respect to t)

$\therefore \frac{dy}{dt} = a b x^{b-1} \frac{dx}{dt}$

Multiply by $\left(\frac{1}{y}\right)$

$\therefore \frac{1}{y} \times \frac{dy}{dt} = \frac{1}{y} \times \frac{a b x^b}{x} \times \frac{dx}{dt}$

$\therefore \frac{1}{y} \times \frac{dy}{dt} = \frac{1}{y} \times \frac{b}{x} \times y \times \frac{dx}{dt}$

$\therefore \frac{1}{y} \times \frac{dy}{dt} = \frac{b}{x} \times \frac{dx}{dt}$

Another solution : (by taking natural logarithm of both sides)

$\therefore \ln y = \ln a + b \ln x$

(by differentiate with respect to t)

$\therefore \frac{1}{y} \times \frac{dy}{dt} = \frac{b}{x} \times \frac{dx}{dt}$

14

The volume

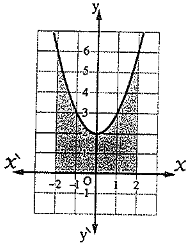
$= \pi \int_{-2}^{2} y^2 \, dx$

$= \pi \int_{-2}^{2} (x^2+2)^2 \, dx$

$= \pi \int_{-2}^{2} (x^4 + 4x^2 + 4) \, dx$

$= \pi \left[\frac{1}{5} x^5 + \frac{4}{3} x^3 + 4x\right]_{-2}^{2}$

$= \frac{752}{15} \pi$ cubic unit.



15 (b) **16** (d)

17

let the radius length of the circle $= r$

, length of arc $= \ell$

$\therefore$ area of sector $= \frac{1}{2} \ell r = 4$

$\therefore \ell = \frac{8}{r}$

let the perimeter of the sector (P)

$P = 2r + \ell = 2r + \frac{8}{r} = 2r + 8r^{-1}$

$\therefore P' = 2 - 8r^{-2} \quad , \quad P'' = 16 r^{-3}$

Put $P' = 0$

$\therefore r = \pm 2$ (negative value is refused)

$\therefore P''(2) = 2$

$\therefore$ P has minimum value at $r = 2$

, $\ell = \frac{8}{r} = \frac{8}{2} = 4$, $\theta^{rad} = \frac{\ell}{r} = \frac{4}{2} = 2^{rad}$

18

$y = 4 - x^2$, $y = x + 2$

by solving the two equations

$\therefore x = 1$ or $x = -2$

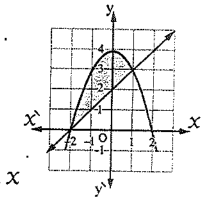
$\therefore$ The area

$= \int_{-2}^{1} \left[(4 - x^2) - (x + 2)\right] dx$

$= \int_{-2}^{1} (2 - x^2 - x) \, dx$

$= \left[2x - \frac{1}{3} x^3 - \frac{1}{2} x^2\right]_{-2}^{1}$

$= \frac{9}{2}$ square unit.



Answers of second session 2018

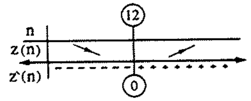
1 ⓐ **2** ⓓ

3

(1) $z(n) = 20\left[\frac{n}{12} - \ln\left(\frac{n}{12}\right)\right] + 30$

$\therefore z'(n) = 20\left[\frac{1}{12} - \frac{12}{n} \times \frac{1}{12}\right] = 20\left(\frac{n-12}{12n}\right)$

$\therefore z'(n) =$ zero when $n = 12$ days

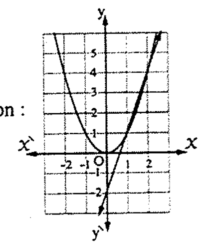


Minimum value after 12 days

(2) The least number of bacteria

$= 20\left[\frac{12}{12} - \ln\left(\frac{12}{12}\right)\right] + 30 = 50$ in each cm^3.

4



$y_1 = x^2$

, $y_2 = 3x - 2$

, to get the point of intersection :

$x^2 = 3x - 2$

$\therefore x^2 - 3x + 2 = 0$

$\therefore x = 1$ or $x = 2$

$\therefore$ The volume $= \pi \int_1^2 (y_2^2 - y_1^2)\, dx$

$= \pi \int_1^2 \left[(3x-2)^2 - (x^2)^2\right] dx$

$= \pi \left[\frac{(3x-2)^3}{3 \times 3} - \frac{1}{5} x^5\right]_1^2$

$= \frac{4}{5} \pi$ cubic unit.

5 ⓒ **6** ⓑ

7

[a] $\int x(x+2)^6\, dx$ putting $z = x + 2$

$\therefore x = z - 2$ $\therefore dx = dz$

$\therefore \int x(x+2)^6\, dx$

$= \int (z-2) z^6\, dz = \int z^7 - 2z^6\, dz$

$= \frac{1}{8} z^8 - \frac{2}{7} z^7 + c$

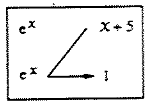
$= \frac{1}{8}(x+2)^8 - \frac{2}{7}(x+2)^7 + c$

[b] $\int (x+5) e^x\, dx$

$= (x+5)e^x - \int e^x\, dx$

$= (x+5)e^x - e^x + c$

$= x e^x + 4 e^x + c$



8 ⓓ **9** ⓑ

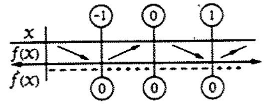
10

[a] $f(x) = x^4 - 2x^2$

$\therefore f'(x) = 4x^3 - 4x$

, $f'(x) = 0$ when $4x(x^2 - 1) = 0$

$\therefore x = 0$ or $x = 1$ or $x = -1$



$\therefore$ The function has a local minimum value

at $x = -1$, $f(-1) = -1$

, the function has a local maximum value at $x = 0$

, $f(0) = 0$

, the function has a local minimum value

at $x = 1$, $f(1) = -1$

[b] $f(x) = \frac{4x}{x^2 + 1}$

$\therefore f'(x) = \frac{(x^2+1) \times (4) - 4x \times 2x}{(x^2+1)^2} = \frac{-4x^2 + 4}{(x^2+1)^2}$

, $f'(x) = 0$ at $x = 1 \in [-1, 3]$

or at $x = -1 \in [-1, 3]$

$\therefore f(-1) = -2$, $f(1) = 2$, $f(3) = \frac{6}{5}$

$\therefore$ The function has absolute maximum value = 2

and absolute minimum value = − 2

11 ⓓ **12** ⓑ

13

$y = 3 + \sec x$ $\therefore \frac{dy}{dx} = \sec x \tan x$

$\therefore \left(\frac{dy}{dx}\right)_{(x = \frac{2\pi}{3})} = 2\sqrt{3}$ $\therefore$ slope of tangent $= 2\sqrt{3}$

, slope of normal $= \frac{-1}{2\sqrt{3}}$

, when $x = \frac{2\pi}{3}$ $\therefore y = 1$

$\therefore$ The point is $\left(\frac{2\pi}{3}, 1\right)$

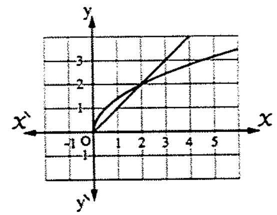
$\therefore$ The equation of tangent : $\frac{y-1}{x - \frac{2\pi}{3}} = 2\sqrt{3}$

i.e. $2\sqrt{3}x - y - \frac{4}{3}\sqrt{3}\pi + 1 = 0$

, equation of normal : $\frac{y-1}{x - \frac{2\pi}{3}} = \frac{-1}{2\sqrt{3}}$

i.e. $x + 2\sqrt{3}y - 2\sqrt{3} - \frac{2}{3}\pi = 0$

14



$y_1 = \sqrt{2x}$, $y_2 = x$

By solving the two

equations $2x = x^2$

$x(x-2) = 0$

$\therefore x = 0$ or $x = 2$

$\therefore$ The area $= \int_0^2 \left(\sqrt{2x} - x\right) dx$

$= \left[\frac{2\sqrt{2}}{3} x^{\frac{3}{2}} - \frac{1}{2} x^2\right]_0^2 = \frac{2}{3}$ square unit.

15 ⓑ **16** ⓓ

17

$\because \sin x = xy$ (Differentiate with respect to x)

$\therefore \cos x = x y' + y$

Differentiate with respect to x another time

$-\sin x = x y'' + y' + y'$ $\therefore -\sin x = x y'' + 2y'$

$\therefore -xy = x y'' + 2 \times \left(\frac{\cos x - y}{x}\right)$

$\therefore -x^2 y = x^2 y'' + 2\cos x - 2y$

$\therefore x^2 (y + y'') + 2\cos x = 2y$

18

$x e^y = 2 - \ln 2 + \ln x$

Differentiate with respect to "t"

$\therefore x e^y \frac{dy}{dt} + e^y \frac{dx}{dt} = \frac{1}{x} \times \frac{dx}{dt}$

$\therefore \frac{dx}{dt} = 6$ at $x = 2$, $y = 0$

$\therefore 2 \times e^0 \times \frac{dy}{dt} + e^0 \times 6 = \frac{1}{2} \times 6$

$\therefore 2 \times \frac{dy}{dt} = -3$

$\therefore \frac{dy}{dt} = -\frac{3}{2}$

Answers of model examinations

Model 1

1 ⓓ **2** ⓐ **3** ⓓ **4** ⓓ

5

$x = e^{2t}$ $\therefore \frac{dx}{dt} = 2e^{2t}$

$y = t^3$ $\therefore \frac{dy}{dt} = 3t^2$

$\frac{dy}{dx} = 3t^2 \div 2e^{2t} = \frac{3t^2}{2e^{2t}}$

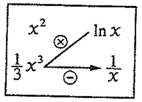
$, \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{3t^2}{2e^{2t}}\right) = \left(\frac{2e^{2t}(6t) - 3t^2 \times 4e^{2t}}{4e^{4t}}\right) \times \frac{dt}{dx}$

$= \frac{12te^{2t}(1-t)}{4e^{4t}} \times \frac{1}{2e^{2t}}$

$= \frac{3}{2} te^{-4t}(1-t)$

6 ⓓ **7** ⓒ

8



[a] $\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x$

$- \int \frac{1}{3} x^3 \times \frac{1}{x} dx$

$= \frac{1}{3} x^3 \ln x - \frac{1}{3} \int x^2 dx$

$= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + c$

$= \frac{1}{9} x^3 [3 \ln x - 1] + c$

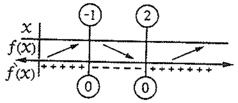
[b] $\int (x^{2e} + e^{3x}) dx = \frac{x^{2e+1}}{2e+1} + \frac{e^{3x}}{3} + c$

9

$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$

$= 6(x+1)(x-2)$

$\therefore f'(x) = 0$ when $x = -1$ or $x = 2$



$\therefore$ Then function is increasing in $]-\infty, -1[$, $]2, \infty[$ and decreasing in $]-1, 2[$ (first req.) $\therefore$ at $x = -1$ there exist a local maximum value $= f(-1) = 19$ at $x = 2$ there exist a local minimum value $= f(2) = -8$ (second req.)

10

Slope of the tangent $= \frac{dy}{dx} = x\sqrt{x^2+1}$

$\therefore y = \int x(x^2+1)^{\frac{1}{2}} dx = \frac{1}{2} \int (2x)(x^2+1)^{\frac{1}{2}} dx$

$= \frac{1}{2} \times \frac{2}{3}(x^2+1)^{\frac{3}{2}} + c = \frac{1}{3}(x^2+1)^{\frac{3}{2}} + c$

$\because$ The point $(0, 1) \in$ the curve

$\therefore 1 = \frac{1}{3}(0+1)^{\frac{3}{2}} + c$ $\therefore c = \frac{2}{3}$

$\therefore$ The equation of the curve : $y = \frac{1}{3}(x^2+1)^{\frac{3}{2}} + \frac{2}{3}$

11 ⓒ **12** ⓓ **13** ⓐ **14** ⓑ

15

Let (x, y) is a point on the curve

$y^2 = 4x$ $\therefore x = \frac{1}{4}y^2$

$\therefore$ The point $\left(\frac{1}{4}y^2, y\right)$

$\therefore$ The distance between the point and the straight line.

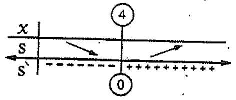
$S = \frac{|x_1 - 2y_1 + 10|}{\sqrt{1^2 + (-2)^2}} = \frac{|\frac{1}{4}y^2 - 2y + 10|}{\sqrt{5}}$

$S = \frac{1}{\sqrt{5}} |\frac{1}{4}y^2 - 2y + 10|$

$\because \frac{1}{4}y^2 - 2y + 10$ is + ve for all values of y

$\therefore S = \frac{1}{\sqrt{5}}\left(\frac{1}{4}y^2 - 2y + 10\right)$ $S' = \frac{1}{\sqrt{5}}\left(\frac{1}{2}y - 2\right)$

Put $S' = 0$ $\therefore y = 4$



The distance is smallest at $y = 4$

$\therefore$ Smallest distance is $S = \frac{6\sqrt{5}}{5}$ length unit

16

The area $= \int_0^2 y \, dx = \int_0^2 x^3 dx$

$= \left[\frac{x^4}{4}\right]_0^2 = \left[\frac{16}{4}\right] - [\text{zero}] = 4$

17

$x^3 + y^3 = 1$ (by differentiate with respect to x)

$\therefore 3x^2 + 3y^2 y' = 0$ (by differentiate with respect to x)

$6x + 3(2y y' \times y' + y^2 y'') = 0$ divided by 3

$y^2 y'' + 2y y'^2 + 2x = 0$

18

$\overline{AB}$ is a tangent , $\overline{MB}$ is a radius

$\therefore \overline{AB} \perp \overline{MB}$ $\therefore \csc\theta = \frac{r + x}{r}$

$\therefore \csc\theta = 1 + \frac{x}{r}$ $\therefore x = r(\csc\theta - 1)$

By differentiation respect to θ

$\frac{dx}{d\theta} = r(-\csc\theta \cot\theta)$

at $\theta = \frac{\pi}{6}$ $\therefore \frac{dx}{d\theta} = -2\sqrt{3}\,r$

Model 2

1 ⓒ **2** ⓒ **3** ⓑ **4** ⓐ

5

Let the dimensions of the rectangle are x and y and the area = A

$\therefore A = xy$

$\because x^2 + y^2 = 64$ $\therefore y^2 = 64 - x^2$

$\therefore y = \sqrt{64 - x^2}$ $\therefore A = x\sqrt{64 - x^2}$

$\therefore \frac{dA}{dx} = x \times \frac{-2x}{2\sqrt{64-x^2}} + \sqrt{64-x^2}$

$= \frac{-x^2}{\sqrt{64-x^2}} + \sqrt{64-x^2}$

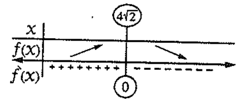
$= \frac{-x^2 + 64 - x^2}{\sqrt{64-x^2}} = \frac{64 - 2x^2}{\sqrt{64-x^2}}$

At the maximum or minimum values

$A' =$ zero

$\therefore 64 - 2x^2 = 0$ $\therefore x = 4\sqrt{2}$

(The negative solution refused)



$\because$ Sign of A' changes from positive to negative at $x = 4\sqrt{2}$

$\therefore$ At $x = 4\sqrt{2}$ the area A has maximum value

$\therefore$ The dimensions of the rectangle are $4\sqrt{2}$, $4\sqrt{2}$

$\therefore$ The area $= 4\sqrt{2} \times 4\sqrt{2} = 32$ cm^2

6

$f(x) = 2x^3 + 3x^2 - 12x + 5$

$\therefore f'(x) = 6x^2 + 6x - 12 = 6(x^2 + x - 2)$

$= 6(x+2)(x-1)$

$f'(x) = 0$ at $x = -2$ or $x = 1$

$\because f''(x) = 12x + 6$ $\therefore f''(-2) < 0$

$\therefore$ There exist a local maximum value $= f(-2) = 25$

$\because f''(1) > 0$

$\therefore$ There exist a local minimum value $= f(1) = -2$

$\because f''(x) = 0$ $\therefore x = -\frac{1}{2}$

$\because f''(x)$ changes in the neighbourhood of $x = -\frac{1}{2}$

$\therefore$ Exist an inflection point at $x = -\frac{1}{2}$

$f\left(-\frac{1}{2}\right) = 11\frac{1}{2}$

$\therefore$ The inflection point is $\left(-\frac{1}{2}, 11\frac{1}{2}\right)$

7 ⓑ **8** ⓒ

9

[a] $y = (\sin x)^{\tan x}$ (take log of each side)

$\therefore \ln y = \tan x \ln(\sin x)$

(by differentiation with respect to x)

$\therefore \frac{y'}{y} = \sec^2 x \ln(\sin x) + \tan x \times \frac{\cos x}{\sin x}$

$= \sec^2 x \ln(\sin x) + 1$

$\therefore \frac{dy}{dx} = (\sin(x))^{\tan x}\left(\sec^2 x \ln(\sin x) + 1\right)$

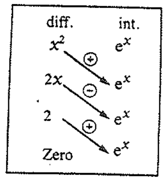
[b] $y = (5 - 3\csc x)^4$

$\frac{dy}{dx} = 4(5 - 3\csc x)^3(3\csc x \cot x)$

$= 12 \csc x \cot x (5 - 3\csc x)^3$

10 ⓓ

11



[a] $\int x^2 e^x dx$

$= x^2 e^x - 2xe^x + 2e^x + c$

[b] $\int x(x+2)^8 dx$

$= \int (x + 2 - 2)(x+2)^8 dx$

$= \int [(x+2)^9 - 2(x+2)^8] dx$

$= \frac{1}{10}(x+2)^{10} - \frac{2}{9}(x+2)^9 + c$

12 ⓒ

13

$\because y^2 + ax^3 - bx = c$ (by differentiation with respect to x)

$\therefore 2y\frac{dy}{dx} + 3ax^2 - b = 0$

(by differentiation with respect to x)

$\therefore 2y\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^2 + 6ax = 0$ (divide by 2)

$\therefore y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + 3ax = 0$

14 ⓑ

15

$\because f'(x) = 6x - 4$ $\qquad \therefore f'(x) = 3x^2 - 4x + c_1$

At $x = 1$, then $f'(x) = 0$

$\therefore 3 \times 1 - 4 \times 1 + c_1 = 0 \qquad \therefore c_1 = 1$

$\therefore f'(x) = 3x^2 - 4x + 1$

$\therefore f(x) = x^3 - 2x^2 + x + c_2$

, $\because$ the curve passes through (1 , 5) :

$\therefore 5 = 1 - 2 + 1 + c_2 \qquad \therefore c_2 = 5$

$\therefore f(x) = x^3 - 2x^2 + x + 5$

16 ⓓ

17

Let the length of two sides of the triangle are L , M and measure of the included angle is θ

$A = \frac{1}{2} L M \sin\theta$

$\frac{dA}{dt} = \frac{1}{2} M \sin\theta \frac{dL}{dt} + \frac{1}{2} L \sin\theta \frac{dM}{dt} + \frac{1}{2} L M \cos\theta \frac{d\theta}{dt}$

at $L = M = 10$, $\theta = \frac{\pi}{3}$, then :

$\frac{dA}{dt} = \frac{1}{2} \times 10 \times \frac{\sqrt{3}}{2} \times 0.1 + \frac{1}{2} \times 10 \times \frac{\sqrt{3}}{2} \times 0.1 + \frac{1}{2} \times 10 \times 10 \times \frac{1}{2} \times \frac{1}{5}$

$\simeq 5.866 \text{ cm}^2/\text{sec.}$

18

Equation of the straight line which passes through the two points (0 , 6) , (1 , 7) :

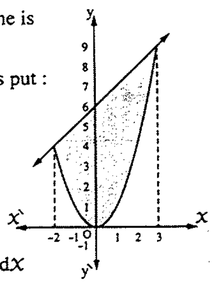
$\because$ The slope $= \frac{7-6}{1-0} = 1$

, the y– intercepted part = 6

$\therefore$ Equation of the straight line is

$y = x + 6$

to find the intersection points put :



$x^2 = x + 6$

$\therefore x^2 - x - 6 = 0$

$\therefore (x-3)(x+2) = 0$

$\therefore x = 3 \quad , \quad x = -2$

$\therefore$ The volume

$= \pi \int_{-2}^{3} \left((x+6)^2 - x^4\right) dx$

$= \pi \int_{-2}^{3} (x^2 + 36 + 12x - x^4) dx$

$= \pi \left[\frac{1}{3} x^3 + 36x + 6x^2 - \frac{1}{5} x^5\right]_{-2}^{3}$

$= \frac{500}{3} \pi$ cubic unit

Model 3

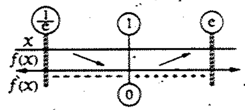
1 ⓐ **2** ⓑ **3** ⓓ **4** ⓓ **5** ⓐ

6

$\because f(x) = x - \ln x$

$f'(x) = 1 - \frac{1}{x}$

$f'(x) = 0$



$x = 1 \in \left[\frac{1}{e}, e\right]$

the function is decreasing in the interval $\left[\frac{1}{e}, 1\right]$ and increasing in the interval $[1, e]$ (1[st] req.)

$f\left(\frac{1}{e}\right) = \frac{1}{e} - \ln\frac{1}{e} = \frac{1}{e} + 1 \simeq 1.37$

$f(1) = 1 - \ln 1 = 1$

$f(e) = e - \ln e = e - 1 \simeq 1.7$

The function has absolute maximum value

$= e - 1$ at $x = e$

and has absolute minimum value = 1 at $x = 1$ (2[nd] req.)

7 ⓒ **8** ⓒ

9

$x^2 y = a b \ln x$ (by differentiation with respect to x)

$\therefore 2xy + x^2 \frac{dy}{dx} = \frac{ab}{x}$ (multiply by x)

$\therefore 2x^2 y + x^3 \frac{dy}{dx} = ab$ (by differentiation with respect to x)

$\therefore 4xy + 2x^2 \frac{dy}{dx} + 3x^2 \frac{dy}{dx} + x^3 \frac{d^2y}{dx^2} = 0$

$\therefore 4xy + 5x^2 \frac{dy}{dx} + x^3 \frac{d^2y}{dx^2} = 0$ (dividing by x)

$\therefore 4y + 5x \frac{dy}{dx} + x^2 \frac{d^2y}{dx^2} = 0$

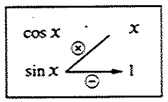
10 ⓓ

11

$\int x \cos x \, dx$

$= x \sin x - \int \sin x \, dx$

$= x \sin x + \cos x + c$



12 ⓓ

13

$\int \sqrt{x}\left(1 + \sqrt{x}\right) dx = \int (x^{\frac{1}{2}} + x) \, dx$

$= \frac{2}{3} x^{\frac{3}{2}} + \frac{1}{2} x^2 + c$

14 ⓓ

15

$f(x) = 2x^3 - 3x^2 - 36x + 14$

$\therefore f'(x) = 6x^2 - 6x - 36$

, put $f'(x) = 0 \qquad \therefore 6(x^2 - x - 6) = 0$

$\therefore 6(x-3)(x+2) = 0$

$\therefore x = 3$ or $x = -2$

, $f''(x) = 12x - 6$

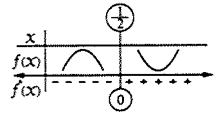
$f''(3) = 30 > 0 \qquad , f(3) = -67$

, $f''(-2) = -30 < 0 \qquad , f(-2) = 58$

The function has local minimum value = – 67 at $x = 3$ and has local maximum value = 58 at $x = -2$ (1[st] req.)

$f''(x) = 0 \qquad \therefore 12x - 6 = 0$

$\therefore x = \frac{1}{2}$



The function is convex upwards in $\left]-\infty, \frac{1}{2}\right[$

and convex downwards in $\left]\frac{1}{2}, \infty\right[$ (2[nd] req.)

16

Area $= \int_{-1}^{2} y \, dx = \int_{-1}^{2} (3x^2 + 4) \, dx$

$= [x^3 + 4x]_{-1}^{2}$

$= [(2)^3 + 4(2)] - [(-1)^3 + 4(-1)]$

= 21 square unit.

17

$\frac{dy}{dx} = \sin x \cos x$ (by integration with respect to x)

$\therefore y = \int (\sin x \cos x) \, dx = \frac{1}{2} \sin^2 x + c$

At $x = \frac{\pi}{6}$, $y = 1 \qquad \therefore 1 = \frac{1}{2} \sin^2 \frac{\pi}{6} + c$

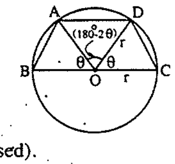
$\therefore c = \frac{7}{8} \qquad \therefore y = \frac{1}{2} \sin^2 x + \frac{7}{8}$

18

Area of trapezium A = surface area of Δ OBA + surface area of Δ OAD + surface area of Δ ODC

$= \frac{1}{2} r^2 \sin\theta + \frac{1}{2} r^2 \sin(180° - 2\theta) + \frac{1}{2} r^2 \sin\theta$

$= r^2 \sin\theta + \frac{1}{2} r^2 \sin 2\theta$



$A' = r^2 \cos\theta + \frac{1}{2} r^2 \cos 2\theta \times 2$

$= r^2 \cos\theta + (2\cos^2\theta - 1) r^2$

$A' = 0$

$\therefore r^2 (2\cos^2\theta + \cos\theta - 1) = 0$

$\cos\theta = \frac{1}{2}$ or $\cos\theta = -1$ (refused).

$\therefore \theta = 60° \qquad \therefore A''_{(\theta = 60°)} < 0$ (Max.)

Then the base angle $m(\angle ABC) = \frac{180° - 60°}{2} = 60°$

Model 4

1 ⓒ **2** ⓐ **3** ⓑ

4 ⓓ **5** ⓑ **6** ⓑ

7

$y^2 = 1 - x^{-2}$ (by differentiation with respect to x)

$\therefore 2y \frac{dy}{dx} = 2x^{-3}$ (÷ 2)

$\therefore y \frac{dy}{dx} = x^{-3}$ (by differentiation with respect to x)

$\therefore y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -3x^{-4}$

$\therefore y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 + \frac{3}{x^4} = 0$

8 ⓐ

9

$f(x) = x e^{-x}$

$\therefore f'(x) = x e^{-x} \times -1 + e^{-x} = -x e^{-x} + e^{-x}$

$= e^{-x}(1 - x)$

put : $f'(x) = 0 \qquad \therefore e^{-x}(1 - x) = 0$

Then $x = 1 \in [0, 2]$

$\because f(0) = $ zero , $f(1) = \frac{1}{e}$, $f(2) = \frac{2}{e^2}$

$\therefore$ The function has absolute maximum value

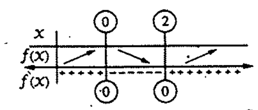
$= \frac{1}{e}$ at $x = 1$

and has absolute minimum value = zero at x = zero

10

$f'(x) = 3x^2 - 6x = 3x(x - 2)$

put $f'(x) = 0 \qquad \therefore x =$ zero , $x = 2$



$\therefore$ The function is increasing in each of $]-\infty$, zero[, $]2, \infty[$ and decreasing in $]0, 2[$ (1[st] req.)

At $x = 0$

exist local maximum value

$\because f(0) = 0$ $\therefore$ The local maximum value = zero

At $x = 2$ exist local minimum value

$\because f(2) = 2^3 - 3(2)^2 = -4$

$\therefore$ The local minimum value $= -4$ (2nd req.)

11 ⓑ

12

[a] Put $z^2 = x - 2$ $\therefore x = z^2 + 2$

$\therefore dx = 2z\,dz$

$\int (x^2+3)\sqrt{x-2}\,dx$

$= \int \left((z^2+2)^2+3\right) \times z \times 2z\,dz$

$= \int 2z^2(z^4+4z^2+7)\,dz$

$= \int (2z^6+8z^4+14z^2)\,dz$

$= \frac{2}{7}z^7 + \frac{8}{5}z^5 + \frac{14}{3}z^3 + c$

$= \frac{2}{7}(x-2)^{\frac{7}{2}} + \frac{8}{5}(x-2)^{\frac{5}{2}} + \frac{14}{3}(x-2)^{\frac{3}{2}} + c$

[b] $\int e^{x^3+2\ln x}\,dx = \int (e^{x^3} \times e^{2\ln x})\,dx$

$= \int e^{x^3} \times x^2\,dx$

$= \frac{1}{3}\int 3x^2 e^{x^3}\,dx$

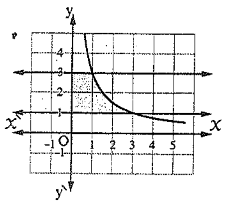
$= \frac{1}{3}e^{x^3} + c$

13 ⓐ

14

$y = \frac{3}{x} = 3x^{-1}$

$\therefore V = \pi \int_0^1 [3^2 - 1^2]\,dx + \pi \int_1^3 (3x^{-1})^2 - (1)^2\,dx$



$= \pi \int_0^1 8\,dx + \pi \int_1^3 (9x^{-2} - 1)\,dx$

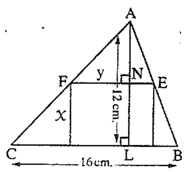
$= \pi[8x]_0^1 + \pi[-9x^{-1} - x]_1^3$

$= 8\pi + 4\pi = 12\pi$ cubic unit.

15

$\Delta\Delta$ AEF , ABC are similar

$\therefore$ The corresponding hieght and corresponding side lengths are proportional



$\therefore \frac{y}{16} = \frac{AN}{AL} = \frac{12-x}{12}$

$\therefore y = 16 - \frac{4}{3}x$

Area of the rectangle $A = xy = 16x - \frac{4}{3}x^2$

$\therefore \acute{A} = 16 - \frac{8}{3}x$, $\acute{A}$ = zero at $x = 6$

$\acute{\acute{A}} = -\frac{8}{3} < 0$ (Max.)

$\therefore$ Dimensions of the rectangle are 6 and 8 cm.

16 ⓒ

17

Slope of the normal $= 3 - x$

$\therefore$ Slope of the tangent $= \frac{-1}{3-x} = \frac{1}{x-3}$

By integration : $\therefore y = \int \frac{1}{x-3}\,dx = \ln|x-3| + c$

$\because A \in$ the curve. $\therefore \ln|2-3| + c = 3$

$\therefore c = 3$

$\therefore$ The equation is $y = \ln|x-3| + 3$

18

Let the base side length = length of hieght $= x$

$\therefore$ Volume of the pyramid $= \frac{1}{3}x^3$ cm^3

$\therefore \frac{dv}{dt} = x^2 \frac{dx}{dt}$

$\therefore 1 = x^2 \times 0.01$

$\therefore x^2 = 100$ $\therefore x = 10$ cm.

$\therefore$ Length of base side length = 10 cm.

Model 5

1 ⓒ **2** ⓐ **3** ⓑ **4** ⓒ

5

$f(x) = x^4 - 2x^2$

$\acute{f}(x) = 4x^3 - 4x = 4x(x^2-1)$

$= 4x(x-1)(x+1)$

put : $\acute{f}(x) = 0$ $\therefore x = 0$ or $x = 1$ or $x = -1$

$\because \acute{\acute{f}}(x) = 12x^2 - 4$

$\because \acute{\acute{f}}(0) < 0$ $\therefore$ At $x = 0$

There exist a local maximum value $= f(0)$ = zero

$\because \acute{\acute{f}}(1) > 0$ $\therefore$ At $x = 1$

There exist local minimum value

$= f(1) = 1 - 2 = -1$

$\because \acute{\acute{f}}(-1) > 0$ $\therefore$ At $x = -1$

There exist a local minimum value $= f(-1) = -1$

6 ⓑ

7

Let $z^3 = 3x + 1$ $\therefore 3x = z^3 - 1$

$3\,dx = 3z^2\,dz$ $\therefore dx = z^2\,dz$

$\int 3x\sqrt[3]{3x+1}\,dx = \int (z^3-1)(z)(z^2)\,dz$

$= \int (z^6 - z^3)\,dz = \frac{1}{7}z^7 - \frac{1}{4}z^4 + c$

$= \frac{1}{7}(3x+1)^{\frac{7}{3}} - \frac{1}{4}(3x+1)^{\frac{4}{3}} + c$

8

$\acute{f}(x) = \frac{x(2x) - (x^2+9)}{x^2} = \frac{2x^2 - x^2 - 9}{x^2}$

$= \frac{x^2-9}{x^2}$ = zero

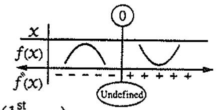
$\therefore x = 3$ or $x = -3$

$\acute{\acute{f}}(x) = \frac{x^2 \times 2x - (x^2-9)(2x)}{x^4} = \frac{2x^3 - 2x^3 + 18x}{x^4}$

$= \frac{18x}{x^4} = \frac{18}{x^3}$

$\therefore$ In the interval $]-\infty, 0[$ the curve convex upward

, in the interval $]0, \infty[$ the curve convex downward (1st req.)



, to find the absolute maximum and absolute minimum in the interval $[1, 6]$

• $f(1) = \frac{1+9}{1} = 10$ • $f(3) = \frac{9+9}{3} = 6$

• $f(6) = \frac{36+9}{6} = 7.5$

, $-3 \notin [1, 6]$

$\therefore$ The absolute maximum in the interval $[1, 6]$ is 10

, absolute minimum in the interval $[1, 6]$ is 6 (2nd req.)

9

[a] $\int \sec^5 x \tan x\,dx = \int \sec^4 x (\sec x . \tan x)\,dx$

$= \frac{1}{5}\sec^5 x + c$

[b] $\int x^2 \sec^2(x^3+5)\,dx$

$= \frac{1}{3}\int \sec^2(x^3+5) \times 3x^2\,dx$

$= \frac{1}{3}\tan(x^3+5) + c$

10

The area $(A) = \int (y_2 - y_1)\,dx$

, finding the intersection points : put $y_2 = y_1$

$\therefore 6 - x^2 = -x$ $\therefore x^2 - x - 6 = 0$

$\therefore (x-3)(x+2) = 0$

$\therefore x = 3$, $x = -2$

$A = \int_{-2}^{3} (6 - x^2) - (-x)\,dx$

$= \int_{-2}^{3} (6 - x^2 + x)\,dx$

$= \left[6x - \frac{1}{3}x^3 + \frac{1}{2}x^2\right]_{-2}^{3}$

$= \left[6(3) - \frac{1}{3}(3)^3 + \frac{1}{2}(3)^2\right]$

$- \left[6(-2) - \frac{1}{3}(-2)^3 + \frac{1}{2}(-2)^2\right]$

$= \frac{125}{6}$ square units.

11 ⓑ **12** ⓓ **13** ⓒ

14

From similarity of the triangles

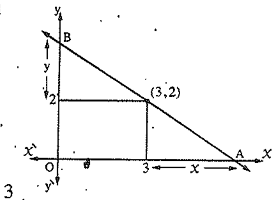
$\frac{x}{3} = \frac{2}{y}$ $\therefore y = \frac{6}{x}$

The area of the triangle AOB

$(A) = \frac{1}{2}(x+3)(y+2)$

$= \frac{1}{2}(x+3)\left(\frac{6}{x}+2\right)$

$= 6 + x + 9x^{-1}$



$\therefore \acute{A} = 1 - 9x^{-2}$

$\therefore \acute{\acute{A}} = 18x^{-3}$

Put $\acute{A} = 0$

then $1 - \frac{9}{x^2} = 0$

$\therefore x^2 = 9$, then $x = 3$

, $\because \acute{\acute{A}}_{at\,x=3} > 0$ minimum value.

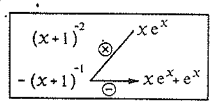
$\therefore$ smallest area $= A_{(at\,x=3)} = 6 + 3 + \frac{9}{3}$

= 12 square units.

15 ⓒ

16

Slope of the tangent $= \frac{dy}{dx}$



$= \frac{xe^x}{(x+1)^2}$

$\therefore y = \int \frac{xe^x}{(x+1)^2}\,dx = \frac{-xe^x}{x+1} - \int \frac{(x+1)e^x}{-(x+1)}\,dx$

$= \frac{-xe^x}{x+1} + e^x + c$

$\because$ The curve passes through $(1, 2e)$,

$\therefore 2e = \frac{-e}{2} + e + c$ $\therefore c = \frac{3}{2}e$

$\therefore$ The equation is : $y = \frac{-xe^x}{x+1} + e^x + \frac{3}{2}e$

17 ⓐ

18

Area of the regular hexagon which side length x :

$A = \frac{6}{4} x^2 \cot \frac{180}{6} = \frac{3}{2}\sqrt{3}\, x^2$

$\frac{dA}{dt} = 3\sqrt{3}\, x \frac{dx}{dt}$

$\left(\frac{dA}{dt}\right)_{x=10} = 3\sqrt{3} \times 10 \times -0.1$

$= -3\sqrt{3}$ cm.^{2}/sec.

Model 6

1 ⓑ **2** ⓑ **3** ⓑ **4** ⓑ

5

Let the point (x , y) lies on the curve $y = \frac{1}{2} x^2 - 4$

$\therefore$ The point is $\left(x , \frac{1}{2} x^2 - 4\right)$

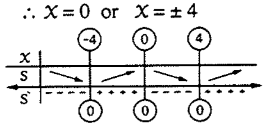
The distance between the two points

$S = \sqrt{(x-0)^2 + \left(\frac{1}{2} x^2 - 4 - 5\right)^2}$

$= \sqrt{\frac{1}{4} x^4 - 8x^2 + 81}$

$\grave{S} = \frac{x^3 - 16x}{2\sqrt{\frac{1}{4} x^4 - 8x^2 + 81}}$

Put $\grave{S} = 0$ $\therefore x = 0$ or $x = \pm 4$



S has minimum value at $x = -4$, 4

$\therefore$ Nearest points are $(-4 , 4)$ and $(4 , 4)$

6 ⓒ

7

Slope of the tangent $= \frac{dy}{dx} = x\sqrt{x+1}$

$\therefore y = \int x\sqrt{x+1}\, dx$

$= \int (x+1-1)(x+1)^{\frac{1}{2}}\, dx$

$= \int (x+1)^{\frac{3}{2}} - (x+1)^{\frac{1}{2}}\, dx$

$= \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + C$

$\therefore \left(0 , \frac{11}{15}\right)$ on the curve.

$\therefore \frac{11}{15} = \frac{2}{5}(0+1)^{\frac{5}{2}} - \frac{2}{3}(0+1)^{\frac{3}{2}} + C$

$\therefore C = 1$

$\therefore$ Equation of the curve

$y = \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + 1$

8 ⓒ

9

$\sqrt{8x} = 6 - x$

$8x = (6-x)^2$

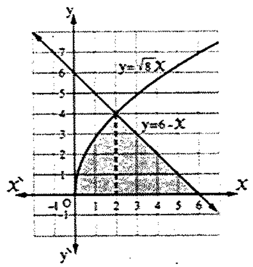
$= 36 - 12x + x^2$

$x^2 - 20x + 36 = 0$

$(x-18)(x-2) = 0$

$\therefore x = 2$

, $x = 18$ (refused)



$V = V_1 + V_2 = \pi\, {}_0\!\int^2 y_1^2\, dx + \pi\, {}_2\!\int^6 y_2^2\, dx$

$= {}_0\!\int^2 8x\, dx + {}_2\!\int^6 (6-x)^2\, dx$

$= [4x^2]_0^2 + \left[\frac{-(6-x)^3}{3}\right]_2^6$

$= \frac{112}{3}\pi$ cubic units

10

$\because y = \sin x(1 + \cos x)$ $\therefore y = \sin x + \sin x \cos x$

$y = \sin x + \frac{1}{2}\sin 2x$ $\therefore \grave{y} = \cos x + \cos 2x$

$, \grave{\grave{y}} = -\sin x - 2\sin 2x$

$\grave{y} = 0 \longrightarrow \cos x + \cos 2x = 0$

$\therefore 2\cos^2 x + \cos x - 1 = 0$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$, $\cos x = \frac{1}{2}$

$\therefore x = \frac{\pi}{3}$ or $\cos x + 1 = 0$

The $\cos x = -1$ (refused).

$\because \grave{\grave{f}}\left(\frac{\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) - 2\sin\left(\frac{2\pi}{3}\right)$

$= \frac{-3\sqrt{3}}{2} < 0$

$\therefore$ At $x = \frac{\pi}{3}$ has local maximum value

$\therefore f\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) + \frac{1}{2}\sin\left(\frac{2\pi}{3}\right)$

$= \frac{\sqrt{3}}{2} + \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{3}{4}\sqrt{3}$

$\therefore$ The local maximum value $= \frac{3}{4}\sqrt{3}$

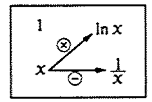
11 ⓓ

12

[a] $\int \ln x\, dx = x\ln x - \int \frac{1}{x} \times x\, dx$

$= x\ln x - \int dx$

$= x\ln x - x + C$



[b] $2\cos^2 x\, dx = \int (1 + \cos 2x)\, dx$

$= x + \frac{1}{2}\sin 2x + C$

13 ⓐ

14

Volume of the sphere $(V) = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

$x = 4\pi r^2 \frac{dr}{dt}$ $\therefore \frac{dr}{dt} = \frac{x}{4\pi r^2}$

Area of the sphere $A = 4\pi r^2$

$\therefore \frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 8\pi r \times \frac{x}{4\pi r^2} = \frac{2x}{r}$ cm^2/sec.

15

Area of the rectangle A

$= (a\cos\theta + b\sin\theta) \times (b\cos\theta + a\sin\theta)$

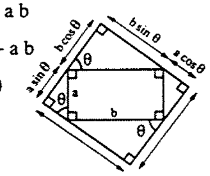
$= ab\cos^2\theta + a^2\sin\theta\cos\theta$

$+ b^2\sin\theta\cos\theta + ab\sin^2\theta$

$= \sin\theta\cos\theta(a^2+b^2) + ab(\sin^2\theta + \cos^2\theta)$

$\therefore A = \sin\theta\cos\theta(a^2+b^2) + ab$

$= (a^2+b^2)\left(\frac{1}{2}\sin 2\theta\right) + ab$



$\frac{dA}{d\theta} = \frac{1}{2}(a^2+b^2) \times 2\cos 2\theta$

Put $\frac{dA}{d\theta} =$ zero

$(a^2+b^2) \cdot \cos 2\theta =$ zero

$\therefore \cos 2\theta =$ zero. $\therefore 2\theta = 90°$

$m(\angle\theta) = 45°$

$\because \frac{d^2A}{d\theta^2} = -2\sin 2\theta(a^2+b^2)$

At $m(\angle\theta) = 45°$

then $\frac{d^2A}{d\theta^2} = -2(a^2+b^2) <$ zero $\therefore m(\angle\theta) = 45°$

The areas has maximum value

$A = \frac{1}{2}(a^2+b^2)\sin 90° + ab$

$2A = a^2 + b^2 + 2ab = (a+b)^2$

$\therefore A = \frac{(a+b)^2}{2}$ square unit.

16 ⓒ **17** ⓐ

18

[a] $\because x^2 + y^2 = 1$ (by differentiation respect to x)

$\therefore 2x + 2y\frac{dy}{dx} = 0$ (divided by 2)

$\therefore x + y\frac{dy}{dx} = 0$ (1)

$\frac{dy}{dx} = \frac{-x}{y}$

By differentiation eq. (1) respect to x

$\therefore 1 + y\frac{d^2y}{dx^2} + \frac{dy}{dx} \times \frac{dy}{dx} = 0$

$\therefore 1 + y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

Substitute by $\frac{dy}{dx} = \frac{-x}{y}$

$\therefore 1 + y\frac{d^2y}{dx^2} + \left(\frac{-x}{y}\right)^2 = 0$

$1 + y\frac{d^2y}{dx^2} + \frac{x^2}{y^2} = 0$ multiply by (y^2)

$\therefore y^2 + y^3\frac{d^2y}{dx^2} + x^2 = 0$

$x^2 + y^2 = 1$ $\therefore y^3\frac{d^2y}{dx^2} + 1 = 0$

[b] $\frac{dy}{dx} = -\sin x$, $\frac{d^2y}{dx^2} = -\cos x$

L.H.S $= (-\sin x)^2 - \cos x.(-\cos x)$

$= \sin^2 x + \cos^2 x = 1 =$ R.H.S

Model 7

1 ⓑ **2** ⓓ **3** ⓓ **4** ⓓ

5

$f(x) = \frac{x^2+1}{x^2+3}$

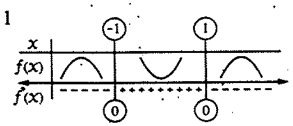
$\grave{f}(x) = \frac{(x^2+3)(2x) - (x^2+1)(2x)}{(x^2+3)^2} = \frac{4x}{(x^2+3)^2}$

$, \grave{\grave{f}}(x) = \frac{(x^2+3)^2 \cdot (4) - 4x\left(2(x^2+3) \times 2x\right)}{(x^2+3)^4}$

$= \frac{(x^2+3)(4x^2 + 12 - 16x^2)}{(x^2+3)^4} = \frac{-12x^2 + 12}{(x^2+3)^3}$

When $\grave{\grave{f}}(x) = 0$, then $-12x^2 + 12 = 0$

$\therefore x = \pm 1$



The curve is convex upward in $]-\infty , -1[$, $]1 , \infty[$ and convex downward in $]-1 , 1[$ and has points of inflection , they are $\left(-1 , \frac{1}{2}\right) , \left(1 , \frac{1}{2}\right)$

6 ⓓ

7

$x^2 y = 2x + 5$ (by differentiation with respect to x)

$\therefore x^2 \frac{dy}{dx} + 2xy = 2$ (by differentiation with respect to x)

$\therefore x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 2x\frac{dy}{dx} + 2y = 0$

$\therefore x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 0$

8 ⓑ **9** ⓐ **10** ⓑ

11

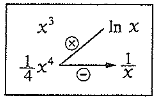
[a] $\int \cot^3 x \, dx = \int \cot^2 x \cot x \, dx$

$= \int (\csc^2 x - 1) \cot x \, dx$

$= \int (\cot x \csc^2 x - \cot x) \, dx$

$= -\frac{1}{2} \cot^2 x + \ln|\csc x| + c$

[b] $\int x^3 \ln x \, dx$



$= \frac{1}{4} x^4 \ln x - \int \frac{1}{4} x^3 \, dx$

$= \frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 + c$

12 ⓑ **13** ⓒ

14

Let radius of its circle $= x$ and length of the arc $= y$

$\therefore$ Its area $= \frac{1}{2} xy = 16$ $\therefore y = \frac{32}{x}$

$\therefore$ The perimeter $P = 2x + y = 2x + \frac{32}{x} = 2x + 32x^{-1}$

$\therefore \dot{P} = 2 - 32x^{-2}$, $\ddot{P} = 64x^{-3}$

put $\dot{P} = 0$ $\therefore x = \pm 4$

, $\ddot{P}(4) = 1$

$\therefore$ P has minimum value at $x = 4$

i.e. its radius length = 4 cm.

, $y = \frac{32}{4} = 8$ cm. $\therefore \theta^{rad} = \frac{y}{x} = \frac{8}{4} = 2^{rad}$

15

[a] Points of intersection : put $y_1 = y_2$

$2x^2 = 3 - x^2$ $\therefore 3x^2 = 3$ $\therefore x = \pm 1$

Area (A) $= \int_{-1}^{1} (y_2 - y_1) \, dx$

$= \int_{-1}^{1} (3 - x^2 - 2x^2) \, dx$

$= \int_{-1}^{1} (3 - 3x^2) \, dx = [3x - x^3]_{-1}^{1}$

$= (3 - 1) - (3(-1) - (-1)^3) = 4$

[b] Point of intersection by solving their equations

$\therefore x = -2$, $x = 1$

$\therefore$ Area $= \int_{-2}^{1} ((2 - x) - x^2) \, dx$

$= \int_{-2}^{1} (2 - x - x^2) \, dx$

$= \left[2x - \frac{1}{2}x^2 - \frac{1}{3}x^3\right]_{-2}^{1}$

$= 4\frac{1}{2}$ square units.

16

Slope of the normal of the curve $= \frac{5 + 2y}{2 - 3x^2}$

$\therefore$ Slope of the tangent $= -\frac{2 - 3x^2}{5 + 2y} = \frac{3x^2 - 2}{5 + 2y}$

$\therefore \frac{dy}{dx} = \frac{3x^2 - 2}{2y + 5}$

$\therefore \int (2y + 5) \, dy = \int (3x^2 - 2) \, dx$

$\therefore y^2 + 5y = x^3 - 2x + c$ where c is constant

$\because$ The curve passes through the point (1 , 2)

$\therefore y = 2$ when $x = 1$

$\therefore 4 + 10 = 1 - 2 + c$ $\therefore c = 15$

$\therefore$ The equation of the curve is : $y^2 + 5y = x^3 - 2x + 15$

17

At $x = 1$ $\therefore 2 + \ln y \ln 1 = 1^2 + y$

$\therefore y = 1$ $\therefore$ the point is (1 , 1)

$\because 2 + \ln y \ln x = x^2 + y$

(by differentiation with respect to x)

$\therefore \ln y \times \frac{1}{x} + \ln x \times \frac{1}{y} \times \frac{dy}{dx} = 2x + \frac{dy}{dx}$

By substitution $x = 1$, $y = 1$

$\therefore \ln(1) \times \frac{1}{1} + \ln 1 \times \frac{1}{1} \times \frac{dy}{dx} = 2(1) + \frac{dy}{dx}$

$\therefore \frac{dy}{dx} = -2$

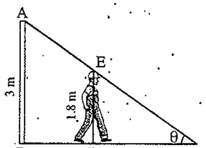
$\therefore$ Equation of the tangent is : $y - 1 = -2(x - 1)$

$\therefore 2x + y - 3 = 0$

Equation of the normal is $y - 1 = \frac{1}{2}(x - 1)$

$\therefore x - 2y + 1 = 0$

18



From the similarity of two triangles ABC , EDC

$\frac{y}{y + x} = \frac{1.8}{3} = \frac{3}{5}$

$3x = 2y$

(by differentiation with respect to t)

$\therefore 3\frac{dx}{dt} = 2\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{3}{2} \times 1.2 = 1.8$ m./sec.

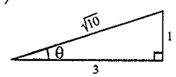
(rate of change of the length of man's shadow)

In ΔABC : $\tan\theta = \frac{3}{x + y} = \frac{3}{x + \frac{3}{2}x} = \frac{6}{5x}$

$\therefore x = \frac{6}{5} \cot\theta$ (by differentiation with respect to t)

$\therefore \frac{dx}{dt} = \frac{6}{5}(-\csc^2\theta)\frac{d\theta}{dt}$

When $x = 3.6$ then $\theta = \tan^{-1}\left(\frac{1}{3}\right)$



$\therefore 1.2 = \frac{6}{5} \times -\left(\frac{\sqrt{10}}{1}\right)^2 \times \frac{d\theta}{dt}$

$\therefore \frac{d\theta}{dt} = \left(\frac{-1}{10}\right)$ rad./sec.

Model 8

1 ⓑ **2** ⓑ **3** ⓒ

4

$f(x) = x + \frac{1}{x}$

$\dot{f}(x) = 1 - \frac{1}{x^2}$

$\dot{f}(x) = 0$ then $x^2 = 1$

$\therefore x = 1$ or $x = -1 \notin \left[\frac{1}{2}, 3\right]$

$\because f(1) = 1 + \frac{1}{1} = 2$

$f\left(\frac{1}{2}\right) = \frac{1}{2} + 2 = 2\frac{1}{2}$

$f(3) = 3\frac{1}{3}$

$\therefore$ The function has absolute maximum value $= 3\frac{1}{3}$ at $x = 3$

and has absolute minimum value = 2 at $x = 1$

5

Area of regular octagon (A) $= \frac{8}{4} x^2 \cot\frac{180}{8}$

$A = 2x^2 \cot\frac{45}{2}$

$\frac{dA}{dt} = 4x \cot\frac{45}{2} \frac{dx}{dt}$

at $x = 10$

$\therefore \frac{dA}{dt} = 4 \times 10 \cot\frac{45}{2} \times 0.2 \simeq 19.3$ cm²/sec.

6 ⓑ **7** ⓒ **8** ⓑ **9** ⓑ

10

$\because f(x) = 1 + 6x - 2x^3$

$\therefore \dot{f}(x) = 6 - 6x^2$, $\ddot{f}(x) = -12x$

$\dot{f}(x) = 0$ $\therefore 6(1 - x^2) = 0$

$\therefore 6(1 - x)(1 + x) = 0$ $\therefore x = \pm 1$

At $x = -1$ then $\ddot{f}(-1) = 12 > 0$

$\therefore$ When $x = -1$ then it has local minimum value

$= f(-1) = 1 + 6(-1) - 2(-1)^3 = -3$

When $x = 1$ then $\ddot{f}(1) = -12 < 0$

$\therefore$ When $x = 1$ it has local maximum value

$f(1) = 1 + 6(1) - 2(1)^2 = 5$ (1st req.)

At $x > 0$ then $\ddot{f}(x) < 0$

i.e. it is convex up. in the interval $]0, \infty[$

and convex down $]-\infty, 0[$

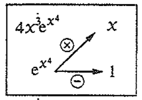
$\therefore$ At $x = 0$ it has inflection point

$\because f(0) = 1 + 6(0) - 2(0)^3 = 1$

$\therefore$ The point of inflection is (0 , 1) (2nd req.)

11

[a] $\int (1 + 4x^4) e^{x^4} \, dx$



$= \int e^{x^4} dx + \int 4x^4 e^{x^4} dx$

$= \int e^{x^4} dx + \int x \times 4x^3 e^{x^4} dx$

$= \int e^{x^4} dx + x e^{x^4} - \int e^{x^4} dx$

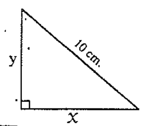
$= x e^{x^4} + C$

[b] $\int x(x + 2)^{-\frac{1}{2}} dx = \int (x + 2 - 2)(x + 2)^{-\frac{1}{2}} dx$

$= \int \left[(x + 2)^{\frac{1}{2}} - 2(x + 2)^{-\frac{1}{2}}\right] dx$

$= \frac{2}{3}(x + 2)^{\frac{3}{2}} - 4(x + 2)^{\frac{1}{2}} + C$

12



$x^2 + y^2 = 100$

$y = \sqrt{100 - x^2}$

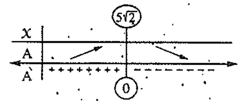
Area of Δ (A) $= \frac{1}{2} xy$

$A = \frac{1}{2} x\sqrt{100 - x^2}$

$\dot{A} = \frac{1}{2} x \times \frac{-x}{\sqrt{100 - x^2}} + \frac{1}{2}\sqrt{100 - x^2}$

$= \frac{-x^2 + 100 - x^2}{2\sqrt{100 - x^2}} = \frac{100 - 2x^2}{2\sqrt{100 - x^2}}$

$\dot{A} = 0$ $\therefore x = 5\sqrt{2}$



When $x = 5\sqrt{2}$ to make A maximum value.

Length of sides of right angle are $5\sqrt{2}$ cm. , $5\sqrt{2}$ cm.

13

$\because \acute{f}(x) = 6x^2 - 30x + 36$

$\therefore \acute{f}(x) = 6(x^2 - 5x + 6)$

$\therefore \acute{f}(x) = 6(x-2)(x-3)$

$\because \acute{f}(x) = 0$ When $x = 2$, $x = 3$

$\therefore f(x)$ has two critical points $x = 2$, $x = 3$

$\because \ddot{f}(x) = 12x - 30$

$\therefore \ddot{f}(x) = 24 - 30 = -6 < 0$

$\therefore (2, f(2))$ is the point of local maximum value

$\because \ddot{f}(3) = 36 - 30 = 6 > 0$

$\therefore (3, f(3))$ is the point of local minimum value.

$\because$ Value of y at the local maximum point = 28

$\therefore f(2) = 28$ $\therefore (2, 28)$ lies on the curve

$\because \acute{f}(x) = 6x^2 - 30x + 36$

$\therefore$ Equation of family of curves is $f(x) = \int \acute{f}(x)\, dx$

i.e. $f(x) = \int (6x^2 - 30x + 36)\, dx$

$f(x) = 2x^3 - 15x^2 + 36x + c$

$\because f(2) = 28$ $\therefore 28 = 16 - 60 + 72 + c$

$c = 0$

$\therefore$ Equation of the required curve :

$y = 2x^3 - 15x^2 + 36x$

14 ⓒ **15** ⓐ

16

$y = e^x \sin x$

$\therefore \frac{dy}{dx} = e^x \cos x - \sin x \times e^x$

$= e^x (\sin x + \cos x)$

$\frac{d^2y}{dx^2} = e^x(\cos x - \sin x) + (\sin x + \cos x) \times e^x$

$= e^x (2\cos x)$

$\text{L.H.S} = \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y$

$= e^x \times 2\cos x - 2 \times e^x(\sin x + \cos x) + 2 \times e^x \sin x$

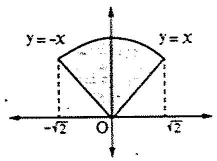
$= 2\cos x \times e^x - 2\sin x \times e^x$

$- 2\cos x \times e^x + 2\sin x \times e^x = \text{zero.}$

17 ⓑ

18

Points of intersection



$4 - x^2 = x^2$

$\therefore 2x^2 = 4$

$x = \pm\sqrt{2}$

$v = \pi \int_{-\sqrt{2}}^{\sqrt{2}} (y_2^2 - y_1^2)\, dx$

$= 2\pi \int_0^{\sqrt{2}} (4 - x^2 - x^2)\, dx$

$= 2\pi \int_0^{\sqrt{2}} (4 - 2x^2)\, dx = 2\pi \left[4x - \frac{2}{3}x^3\right]_0^{\sqrt{2}}$

$= \frac{16\sqrt{2}}{3}\pi$ cubic unit.

Model 9

1 ⓐ

2

$y = a\cos(\ln x) + b\sin x(\ln x)$

(by differentiation with respect to x)

$\frac{dy}{dx} = \frac{-a\sin(\ln x)}{x} + \frac{b\cos(\ln x)}{x}$ $(\times x)$

$x\frac{dy}{dx} = -a\sin(\ln x) + b\cos(\ln x)$

(by differentiation with respect to x)

$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{-a\cos(\ln x)}{x} - \frac{b\sin(\ln x)}{x}$ $(\times x)$

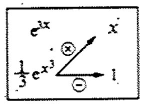
$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -\left(a\cos(\ln x) + b\sin(\ln x)\right)$

$\therefore x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

3 ⓑ **4** ⓒ

5

$\frac{dy}{dx} = xe^{3x}$



$\therefore y = \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}\, dx$

$y = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c$

$\therefore \left(\frac{1}{3}, 5\right)$ lies on the curve

$\therefore 5 = \frac{1}{3}\left(\frac{1}{3}\right)e - \frac{1}{9}e + c$ $\therefore c = 5$

$\therefore y = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + 5$

6 ⓒ **7** ⓑ **8** ⓐ **9** ⓓ

10

[a] $\int \sqrt{\frac{1+\sqrt{x}}{x}}\, dx = \int \frac{(1+\sqrt{x})^{\frac{1}{2}}}{\sqrt{x}}\, dx$

$= 2\int \frac{1}{2\sqrt{x}}(1+\sqrt{x})^{\frac{1}{2}}\, dx$

$= 2 \times \frac{(1+\sqrt{x})^{\frac{3}{2}}}{\frac{3}{2}} + C$

$= \frac{4}{3}(1+\sqrt{x})^{\frac{3}{2}} + C$

[b] $\int (\sin^2 x + \cos^2 x + \cot^2 x)\, dx$

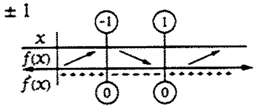
$= \int (1 + \cot^2 x)\, dx = \int \csc^2 x\, dx = -\cot x + C$

11

$y = x^3 - 3x + 4$

$\therefore \acute{y} = 3x^2 - 3$, $\ddot{y} = 6x$

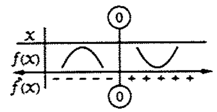
* put $\acute{y} = 0$ $\therefore x = \pm 1$



$\therefore f$ is increasing in $]-\infty, -1[\, ,]1, \infty[$

and decreasing in $]-1, 1[$ (1[st] req.)

* Put $\ddot{y} = 0$ $\therefore x = 0$

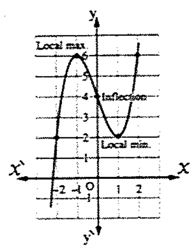


* To find intersection points with y-axis

Put $x = 0$, $y = 4$

* Determine some assistant points $f(-2) = 2$, $f(2) = 6$

x	-2	-1	0	1	2
$f(x)$	2	6	4	2	6



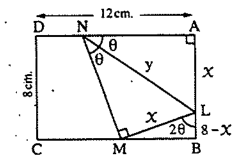
(2[nd] req.)

12 ⓑ

13

In Δ LBM :

$\cos 2\theta = \frac{8-x}{x}$ (1)



In Δ ALN :

$\sin\theta = \frac{x}{y}$

$\because \cos 2\theta = 1 - 2\sin^2\theta$

$= 1 - 2\left(\frac{x}{y}\right)^2$ (2)

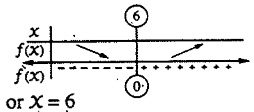
From (1) , (2) : $\frac{8-x}{x} = 1 - 2\left(\frac{x}{y}\right)^2$

$\therefore y^2 = \frac{x^3}{x-4}$ (by differentiation with respect to x)

$2y\frac{dy}{dx} = \frac{(x-4)(3x^2) - x^3(1)}{(x-4)^2}$

$= \frac{2x^3 - 12x^2}{(x-4)^2}$

Put $\frac{dy}{dx} = 0$



$\therefore x = 0$ (refused.) or $x = 6$

$\therefore x = 6$ Makes y is minimum value.

14 ⓒ **15** ⓒ

16

$y = \sin(\csc 3x^2) + 1$

$\frac{dy}{dx} = \cos(\csc 3x^2) \times -\csc 3x^2 \cot 3x^2$

$\times (6x) + \text{zero}$

$= -6x\cos(\csc 3x^2)\csc 3x^2 \cot 3x^2$

17

Points of intersection with x-axis : $x^2 - 9 = 0$

$\therefore x = 3$, $x = -3$

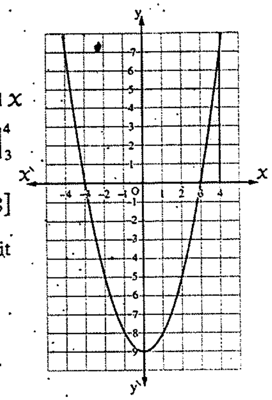
Area $= \int_3^4 y\, dx$

$= \int_3^4 (x^2 - 9)\, dx$

$= \left[\frac{1}{3}x^3 - 9x\right]_3^4$

$= \left[\frac{-44}{3}\right] - [-18]$

$= \frac{10}{3}$ square unit



18

$\frac{dV}{dt} = -20$ cm³/sec.

$\because V = \frac{4}{3}\pi r^3$ $\therefore \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

when r = 10

$-20 = 4\pi \times 10^2 \times \frac{dr}{dt}$ $\therefore \frac{dr}{dt} = \frac{-1}{20\pi}$ cm./sec.

Area (A) $= 4\pi r^2$

$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$\left(\frac{dA}{dt}\right)_{r=10} = 8\pi \times 10 \times \frac{-1}{20\pi} = -4$ cm²/sec.

Model 10

1 ⓒ **2** ⓑ **3** ⓐ

4

$f'(x) = \frac{(x^2+1) - 2x^2}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$

$\because f'(x) = 0$ $\therefore 1 - x^2 = 0$

$x = 1 \in [0, 2]$ or $x = -1 \notin [0, 2]$

$\therefore f(1) = \frac{1}{2}$, $f(0) =$ zero , $f(2) = \frac{2}{5}$

$\therefore$ The absolute maximum value $= \frac{1}{2}$,

The absolute minimum value = zero

5 ⓑ

6

$\frac{dy}{dx} = 7 - \sin 2x$ (by integration with respect to x)

$y = 7x + \frac{1}{2}\cos 2x + c$

$5 = 7(0) + \frac{1}{2}\cos 0 + c$ $\therefore c = \frac{9}{2}$

$\therefore y = 7x + \frac{1}{2}\cos 2x + \frac{9}{2}$

7 ⓑ **8** ⓐ

9

$f(x) = x^3 - 3x + 2$

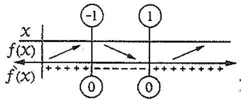
$f'(x) = 3x^2 - 3$

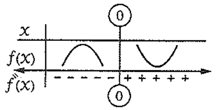
$f''(x) = 6x$

* Put $f'(x) = 0$

$\therefore x = 1$ or $x = -1$

$\because f''(x) = 6x$





Put $f''(x) = 0$ $\therefore x = 0$

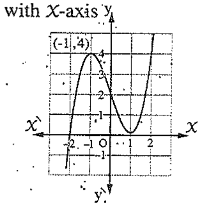
To find points of intersection with x-axis

put $f(x) = 0$

$\therefore x = -2$, $x = 1$

To find points of intersection with y-axis

put $x = 0$ $\therefore y = 2$



10

[a] $\int \frac{\ln x}{x} dx = \int \ln x \times \frac{1}{x} dx = \frac{(\ln x)^2}{2} + C$

[b] $\int x(x^2+3)^5 dx = \frac{1}{2}\int 2x(x^2+3)^5 dx$

$= \frac{1}{12}(x^2+3)^6 + C$

11

[a] $x = a\sec^2\theta$

$\therefore \frac{dx}{d\theta} = 2a\sec\theta . \sec\theta . \tan\theta = 2a\sec^2\theta\tan\theta$

$y = a\tan^3\theta$ $\therefore \frac{dy}{d\theta} = 3a\tan^2\theta . \sec^2\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta} = \frac{3a\tan^2\theta\sec^2\theta}{2a\sec^2\theta\tan\theta} = \frac{3\tan\theta}{2}$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}\left(\frac{dy}{dx}\right) \times \frac{d\theta}{dx}$

$= \frac{3}{2}\sec^2\theta \times \frac{1}{2a\sec^2\theta\tan\theta}$

$= \frac{3\cot\theta}{4a}$

[b] $y = x\tan x$ (by differentiation with respect to x)

$\frac{dy}{dx} = x\sec^2 x + \tan x$

(by differentiation another time)

$\frac{d^2y}{dx^2} = x \times 2\sec x . \sec x\tan x + \sec^2 x + \sec^2 x$

$= 2x\sec^2 x\tan x + 2\sec^2 x$

$= 2\sec^2 x(1 + x\tan x)$

$= 2(1 + y)\sec^2 x$

12

$I = 2\cos t + 2\sin t$

$\frac{dI}{dt} = -2\sin t + 2\cos t$

Put $\frac{dI}{dt} = 0$, $\therefore -2\sin t + 2\cos t = 0$

$\therefore \sin t = \cos t$ $\therefore t = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

$\frac{d^2I}{dt^2} = -2\cos t - 2\sin t$

when $t = \frac{\pi}{4}$

$\frac{d^2I}{dt^2} = -2\sqrt{2} < 0$ (Max)

when $t = \frac{5\pi}{4}$: $\frac{d^2I}{dt^2} = 2\sqrt{2} > 0$ (Min)

$\therefore$ maximum value of current at $t = \frac{\pi}{4}$

is $I = 2\sqrt{2}$

13

Let length of the ladder $= \ell$ unit of length

$x^2 + y^2 = \ell^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\frac{dx}{dt} = \frac{-y}{x} \times \frac{dy}{dt}$

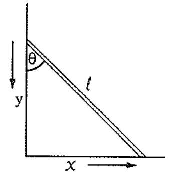
When $\csc\theta = \frac{5}{4}$

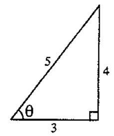
$x = \ell\sin\theta$

, $y = \ell\cos\theta$

$\therefore \frac{dx}{dt} = \frac{-3}{4} \times -k$

$= \frac{3k}{4}$ unit./sec. (k > 0)





14 ⓓ **15** ⓓ **16** ⓑ

17

Find equation of the tangent at (2 , 2)

$\frac{dy}{dx} = \frac{2}{2\sqrt{x-1}} = \frac{1}{\sqrt{x-1}}$

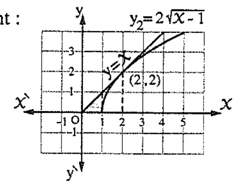
at $x = 2$: $\frac{dy}{dx} = 1$

$\therefore$ Equation of the tangent :

$y - 2 = x - 2$

$\therefore y = x$

$\therefore$ Points of intersection of the tangent with x-axis (0 , 0)



$\therefore v = v_1 + v_2 = \pi\int_0^1 y_1^2 dx + \pi\int_1^2 (y_1^2 - y_2^2) dx$

$= \pi\int_0^1 x^2 dx + \pi\int_1^2 \left[x^2 - \left(2\sqrt{x-1}\right)^2\right] dx$

$= \pi\left[\frac{x^3}{3}\right]_0^1 + \pi\int_1^2 [x^2 - 4(x-1)] dx$

$= \pi\left[\frac{1}{3}\right] + \pi\int_1^2 (x^2 - 4x + 4) dx$

$= \frac{\pi}{3} + \pi\left[\frac{1}{3}x^3 - 2x^2 + 4x\right]_1^2$

$= \frac{\pi}{3} + \pi\left[\left(\frac{8}{3}\right) - \left(\frac{7}{3}\right)\right] = \frac{2\pi}{3}$ cubic unit

18 ⓐ

Model 11

1 ⓑ

2

$\frac{d^2y}{dx^2} = 6 - 12x$

$\therefore \frac{dy}{dx} = \int(6 - 12x) dx$:

$= 6x - 6x^2 + C_1$

$\because$ has critical point at $x = 1$

$\therefore$ zero $= 6 - 6 + C_1$ $\therefore C_1 = 0$

$\frac{dy}{dx} = 6x - 6x^2$ (1)

Put $\frac{dy}{dx} =$ zero $\therefore x = 0$ or 1

At $x = 1$

then $\frac{d^2y}{dx^2} < 0$ has maximum value

At $x = 0$ then $\frac{d^2y}{dx^2} > 0$ has minimum value = 4

$\therefore$ (0 , 4) lies on the curve.

$\because y = \int(6x - 6x^2) dx$

$= 3x^2 - 2x^3 + C_2$

$\therefore 4 = 3(0) - 2(0) + C_2$ $\therefore C_2 = 4$

$\therefore y = 3x^2 - 2x^3 + 4$

$\therefore$ Slope of the tangent at $x = -1$

$\left(\frac{dy}{dx}\right)_{x=-1} = 6(-1) - 6(-1)^2 = -12$

$\therefore$ Slope of the normal $= \frac{1}{12}$ at $x = -1$

$\therefore y = 3(-1)^2 - 2(-1)^3 + 4 = 9$

$\therefore$ The point is (− 1 , 9)

$\therefore$ Equation of the normal $\frac{y-9}{x+1} = \frac{1}{12}$

$\therefore x - 12y + 109 = 0$

3 ⓐ **4** ⓑ **5** ⓐ **6** ⓑ

7

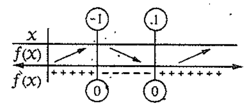
$f(x) = (x+1)(x^2 - x - 2)$

$\therefore f'(x) = (x+1)(2x - 1) + (x^2 - x - 2)$

$= 3x^2 - 3$

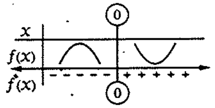
$f''(x) = 6x$

put $f'(x) = 0$ $\therefore x = \pm 1$



$\therefore$ There exist local maximum value $f(-1) =$ zero and exist local minimum value $f(1) = -4$ (1st req.)

put $\acute{f}(x) = 0$ $\therefore x = 0$



$\therefore$ There is inflection point $(0, -2)$

* To find the point of intersection with x-axis

put $f(x) = 0$

$\therefore x = -1, 2$

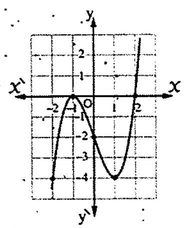
* To find the point of intersection with y-axis

put $x = 0$

$\therefore f(0) = -2$

* Determine other point $f(-2) = -4$

x	-2	-1	0	1	2
$f(x)$	-4	0	-2	-4	0



(2nd req.)

8

$A = \frac{1}{2}$ a b $= 24$ cm^2

zero $= \frac{1}{2}$ a $\frac{db}{dt} + \frac{1}{2}$ b $\frac{da}{dt}$

$\frac{da}{dt} = \frac{-a}{b} \times \frac{db}{dt}$

at b = 8 cm. , then a = 6 cm.

$\frac{da}{dt} = \frac{-6}{8} \times 1 = \frac{-3}{4}$ cm./sec.

$\tan A = \frac{a}{b}$ $\therefore b \tan A = a$

$\frac{db}{dt} \tan A + b \sec A \frac{da}{dt} = \frac{da}{dt}$

$1 \times \frac{3}{4} + 8 \times \left(\frac{5}{4}\right)^2 \times \frac{da}{dt} = \frac{-3}{4}$

$\therefore \frac{da}{dt} = \frac{-3}{25}$ rad./sec. $= -0.12$ rad./sec.

9

$\because y = \sec x$, $\therefore \frac{dy}{dx} = \sec x \tan x$

$\frac{d^2y}{dx^2} = \sec x \sec^2 x + \sec x \tan x \tan x$

$= \sec^3 x + \sec x \tan^2 x$

$= \sec x (\sec^2 x + \tan^2 x)$

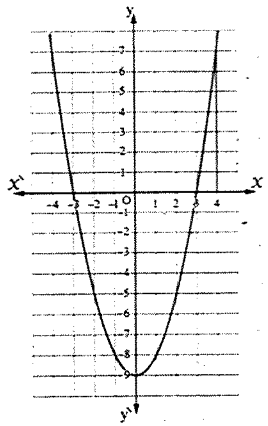
$y \frac{d^2y}{dx^2} + (\frac{dy}{dx})^2 = \sec x \sec x (\sec^2 x + \tan^2 x) + \sec^2 x \tan^2 x$

$= \sec^2 x (\sec^2 x + 2 \tan^2 x)$

$= \sec^2 x (3 \sec^2 x - 2) = y^2 (3 y^2 - 2)$

10 ⓒ

11



To find points of intersection with x-axis

$x^2 - 9 = 0$ $\therefore x = \pm 3$

$\therefore \int_3^4 (x^2 - 9)\, dx = [\frac{x^3}{3} - 9x]_3^4 = \frac{10}{3}$

$\therefore$ Area $= \frac{10}{3}$ square unit.

12 ⓓ **13** ⓒ **14** ⓐ

15

[a] $\int \frac{\cos^2 x}{1 - \sin x} dx = \int \frac{1 - \sin^2 x}{1 - \sin x} dx$

$= \int \frac{(1 - \sin x)(1 + \sin x)}{(1 - \sin x)} dx$

$= \int (1 + \sin x)\, dx = x - \cos x + C$

[b] $\int \frac{1}{x \ln x^3} dx = \int \frac{\frac{1}{x}}{3 \ln x} dx$

$= \frac{1}{3} \int \frac{\frac{1}{x}}{\ln x} dx = \frac{1}{3} \ln |\ln x| + C$

16 ⓒ

17

Let length of its base $= x$, its height = h

$\therefore 8x + 4h = 240$

$\therefore 2x + h = 60$

$\therefore h = 60 - 2x$ (1)

$\because V = x^2 h$

From (1) : $\therefore V = 60 x^2 - 2 x^3$

$\acute{V} = 120 x - 6 x^2$

at $\acute{V} = 0$ $\therefore x = 0$ (refused) or $x = 20$

$\because \acute{\acute{V}} = 120 - 12x$, at $x = 20$

Then $\acute{\acute{V}} = -120 < 0$ $\therefore$ has max. value

$\therefore$ at $x = 20$ cm. then the volume max.

From (1) : $\therefore$ h = 20 cm.

$\therefore$ dimensions of the cuboid which has maximum volume are 20 , 20 , 20 cm.

18

Volume $= \pi \int_{-1}^{k} y^2\, dx = \pi \int_{-1}^{k} (e^x)^2\, dx$

$= \pi \int_{-1}^{k} e^{2x}\, dx = \frac{\pi}{2} [e^{2x}]_{-1}^{k}$

$\therefore \frac{\pi}{2} [e^{2k} - e^{-2}] = \frac{\pi}{2} (e^{10} - e^{-2})$

$\therefore 2k = 10$ $\therefore k = 5$

Model 12

1 ⓑ **2** ⓑ **3** ⓒ **4** ⓒ

5

$\frac{d^2y}{dx^2} = a x + b$

$\because$ The curve has point of inflection at $x =$ zero

$\therefore \frac{d^2y}{dx^2} =$ zero , $\therefore$ zero = zero + b

$\therefore b = 0$

$\therefore \frac{dy}{dx} = \int a x\, dx = \frac{a}{2} x^2 + c$ (c is constant)

$\therefore$ has L.Min. value at (1 , 0)

$\therefore \frac{dy}{dx} =$ zero , at $x = 1$

$\therefore 0 = \frac{a}{2} + c$, then a = − 2 c (1)

$y = \int (\frac{a}{2} x^2 + c)\, dx$

$= \frac{a}{6} x^3 + c x + d$

$\because$ The curve passes through the points (0 , 2) , (1 , 0)

$\therefore 2 = \frac{a}{6}(0) + c(0) + d$ $\therefore d = 2$

$0 = \frac{a}{6}(1) + c(1) + 2$

$\therefore a + 6c = -12$ (2)

by substitute from (1) in (2) :

$\therefore (-2c) + 6c = -12$ $\therefore 4c = -12$

$\therefore c = -3$ a = 6

$\therefore$ equation of the curve $y = x^3 - 3x + 2$

6

$y = \frac{1}{3} x^3 - 9x + 2$

$\therefore \acute{y} = x^2 - 9$

At $\acute{y} = 0$ $\therefore x^2 - 9 = 0$

$\therefore x = 3$ or $x = -3$

$\because \acute{\acute{y}} = 2x$ at $x = 3$ then $\acute{\acute{y}} > 0$

$\therefore$ has local minimum value $= f(3) = -16$

at $x = -3$ then $\acute{\acute{y}} < 0$

$\therefore$ has local minimum value $= f(-3) = 20$

7

$\therefore e^{xy} = x^2 + y$

$\therefore e^{xy} (y + x\acute{y}) = 2x + \acute{y}$

$\therefore y e^{xy} + x \acute{y} e^{xy} = 2x + \acute{y}$

$\acute{y} (x e^{xy} - 1) = 2x - y e^{xy}$

8 ⓒ

9

[a] $\int e^x (\cot x - \csc^2 x)\, dx$

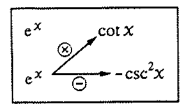
$= \int e^x \cot x\, dx$

$- \int e^x \csc^2 x\, dx$

$= e^x \cot x + \int e^x \csc^2 x\, dx$

$- \int e^x \csc^2 x\, dx$

$= e^x \cot x + C$



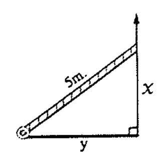
[b] $\int (13)^x\, dx = \frac{(13)^x}{\ln(13)} + C$

10

$\frac{dx}{dt} = 1$ m./min.

y is length of projection of the rod on the ground

$x^2 + y^2 = 25$



$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = \text{zero}$

$\frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$

When $x = 3$ m. , $y = 4$ m.

$\therefore \frac{dy}{dt} = \frac{-3}{4} \times 1 = \frac{-3}{4}$ m./min.

11 ⓓ **12** ⓑ **13** ⓓ **14** ⓐ **15** ⓑ

16

$y_1 = 12 - x^2$, $0 < x < 2\sqrt{3}$

$y_2 = x^2 - 12$. $0 < x < 2\sqrt{3}$

$\therefore$ Dimensions of the rectangle $= 2x$ horizontally.

and vertically $y_1 - y_2 = (12 - x^2) - (x^2 - 12)$

$= 12 - x^2 + 12 - x^2$

$= 24 - 2x^2$

$\therefore$ area of the rectangle $= A = 2x(24 - 2x^2)$

$= 48x - 4x^3$

$\acute{A} = 48 - 12x^2$

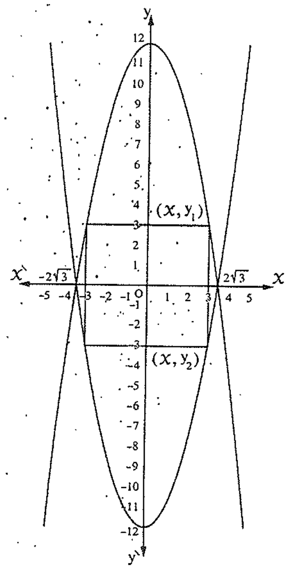
Put $\acute{A} = 0$, $x^2 = 4$, $\therefore x = 2$

$\acute{\acute{A}} = -24x$

$\therefore \acute{\acute{A}}_{(x=2)} < 0$ at $x = 2$ then A has maximum value.

$\therefore$ maximum area of the triangle A

$= 48 \times 2 - 4 \times 2^3 = 64$



17

$f(x) = \sqrt[3]{x-3} = (x-3)^{\frac{1}{3}}$

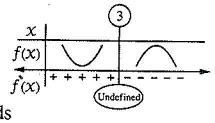
$\therefore \acute{f}(x) = \frac{1}{3}(x-3)^{\frac{-2}{3}}$

$\therefore \acute{\acute{f}}(x) = \frac{-2}{9}(x-3)^{\frac{-5}{3}} = \frac{-2}{9} \times \frac{1}{\sqrt[3]{(x-3)^5}}$

$\acute{\acute{f}}(x)$ undefined when

$x - 3 = 0$

$\therefore x = 3$



The curve is convex upwards in $]3, \infty[$

The curve is convex downwards in $]-\infty, 3[$

$\because$ Sign of $\acute{\acute{f}}(x)$ changes before and after $x = 3$

$\therefore$ It has inflection point at $x = 3$

$f(3) = \text{zero}$

$\therefore$ Inflection point is $(3, 0)$

18

[a] Intersection points : $\sqrt{x} = x$

$\therefore x^2 = x$ $\therefore x^2 - x = 0$

$\therefore x(x-1) = 0$ $\therefore x = 0$ or $x = 1$

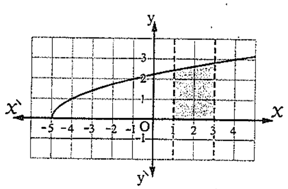
$V = \pi \int_0^1 (y_2^2 - y_1^2)\, dx$

$= \pi \int_0^1 (\sqrt{x})^2 - (x)^2\, dx$

$= \pi \int_0^1 (x - x^2)\, dx = \pi \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= \pi \left[\frac{1}{2} - \frac{1}{3}\right] = \frac{\pi}{6}$ cubic units.

[b]



The volume $= \pi \int_1^3 y^2\, dx = \pi \int_1^3 (x + 5)\, dx$

$= \left[\frac{1}{2}x^2 + 5x\right]_1^3$

$= 14\pi$ cubic unit.

Model 13

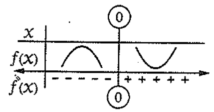
1 ⓓ **2** ⓑ **3** ⓓ **4** ⓒ

5

$y = x(x^2 - 1) = x^3 - x$

$\acute{y} = 3x^2 - 1$ $\acute{\acute{y}} = 6x$

Put $\acute{\acute{y}} = 0$ then $6x = 0$ $\therefore x = 0$



$\therefore$ The curve convex upwards in $]-\infty, 0[$

and convex downwards in $]0, \infty[$

inflection point is $(0, 0)$

6 ⓑ **7** ⓓ

8

[a] $\int (\sin x + \cot x)^9 (\cos x - \csc^2 x)\, dx$

$= \frac{1}{10}(\sin x + \cot x)^{10} + C$

[b] $\frac{1}{2}\int \frac{2x - 1}{\sqrt{2x-1}}\, dx = \frac{1}{2}\int (2x-1)^{\frac{1}{2}}\, dx$

$= \frac{1}{6}(2x-1)^{\frac{3}{2}} + C$

9

$f(x) = \begin{cases} x^2 - 2x & x \geq 2 \\ -x^2 + 2x & x < 2 \end{cases}$

$\therefore \acute{f}(2^+) = \lim_{h \to 0^+} \frac{(2+h)^2 - 2(2+h)}{h} = 2$

$\acute{f}(2^-) = \lim_{h \to 0^-} \frac{(2+h)^2 + 2(2+h)}{h} = -2$

$\acute{f}(2^-) \neq \acute{f}(2^+)$

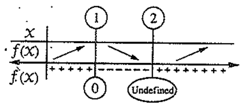
$\therefore$ The function is not differentiable.

At $x = 2$

$\therefore \acute{f}(x) = \begin{cases} 2x - 2 & x > 2 \\ \text{not exist} & x = 2 \\ -2x + 2 & x < 2 \end{cases}$

$\therefore \acute{f}(x) = 0$ at $x = 1$

undefined at $x = 2$



$\therefore$ The function is increasing in $]-\infty, 1[$, $]2, \infty[$

and decreasing in $]1, 2[$ (1[st] req.)

$f\left(\frac{1}{2}\right) = \frac{3}{4}$, $f(1) = 1$

$f(2) = \text{zero}$, $f\left(\frac{5}{2}\right) = \frac{5}{4}$

$\therefore$ Absolute maximum value $= \frac{5}{4}$

absolute minimum value = zero (2[nd] req.)

10 ⓒ

11

$\because v = IR$ (by differentiation respect to t)

$\frac{dv}{dt} = I\frac{dR}{dt} + R\frac{dI}{dt}$ (1)

$\frac{dv}{dt} = 1$ volt/sec. , $\frac{dI}{dt} = \frac{-1}{2}$ Amper/sec.

at $v = 12$ volt , $I = 2$ Amper

$\therefore R = \frac{v}{t} = \frac{12}{2} = 6$ ohm.

By substitution in (1)

$1 = 2 \times \frac{dR}{dt} + 6 \times \frac{-1}{2}$

$\therefore \frac{dR}{dt} = 2$ ohm/sec.

12

$y = 2\sin x - x\cos x$

$\therefore \frac{dy}{dt} = 2\cos x - (x \times -\sin x + \cos x \times 1)$

$= 2\cos x + x\sin x - \cos x$

$= \cos x + x\sin x$

$\therefore \frac{d^2y}{dx^2} = -\sin x + x\cos x + \sin x$

$= x\cos x$

$\therefore \frac{d^2y}{dx^2} + y = x\cos x + 2\sin x - x\cos x = 2\sin x$

13 ⓓ

14

To fined points of intersection

$(2x^2)^2 = 4ax$

$\therefore 4x^4 = 4ax$

$\therefore 4x^4 - 4ax = 0$

$4x(x^3 - a) = 0$

$x = 0$ or $x = \sqrt[3]{a}$

$\int_0^{\sqrt[3]{a}} (y_2 - y_1)\, dx = \int_0^{\sqrt[3]{a}} \left(\sqrt{4ax} - 2x^2\right) dx$

$= \int_0^{\sqrt[3]{a}} \left(2a^{\frac{1}{2}}x^{\frac{1}{2}} - 2x^2\right) dx$

$= \left[\frac{4}{3}a^{\frac{1}{2}}x^{\frac{3}{2}} - \frac{2}{3}x^3\right]_0^{\sqrt[3]{a}}$

$= \frac{4}{3}a - \frac{2}{3}a = \frac{2}{3}a$ $\therefore \frac{2}{3}a = \frac{2}{3}$ $\therefore a = 1$

15 ⓑ   16 ⓐ

17

Let the number of extra appliances = x

$\therefore$ The total number of appliances = $(x + 80)$

$\therefore$ The profit of each appliance = $(50 - \frac{1}{2}x)$

$\therefore$ The total profit (P) = $(x + 80)(50 - \frac{1}{2}x)$

$\therefore P = -\frac{1}{2}x^2 + 10x + 4000$

$\therefore \frac{dP}{dx} = -x + 10$

, at the maximum or minimum values , put $\frac{dP}{dx} = 0$

$\therefore -x + 10 = 0 \qquad \therefore x = 10$

, $\because \frac{d^2P}{dx^2} = -1$ (negative)

$\therefore$ At $x = 10$, the total profit (P) is maximum.

$\therefore$ The total number of the appliances = 10 + 80
= 90 appliance.

18

$y_1 = \frac{4}{x}$, $y_2 = 5 - x$ intersected at $x = 1$

$\therefore y = 4$

$y_1 \le y_2$ in this interval

$\therefore \text{Volume} = \pi \int_1^4 (y_2^2 - y_1^2)\,dx$

$= \pi \int_1^4 (5 - x)^2 - \left(\frac{4}{x}\right)^2 dx$

$= \pi \int_1^4 (25 - 10x + x^2 - 16x^{-2})\,dx$

$= \pi \left[25x - 5x^2 + \frac{1}{3}x^3 + 16x^{-1}\right]_1^4$

$= 9\pi$ cubic unit

Model 14

1 ⓑ   2 ⓓ   3 ⓒ

4

$A = \frac{dy}{dx} = 2x + \frac{1}{2}\sec^2\frac{x}{2}$

$dy = \int (2x + \frac{1}{2}\sec^2\frac{x}{2})\,dx$

$y = x^2 + \tan\frac{x}{2} + C$

at $x = \frac{\pi}{2}$, $y = \frac{\pi^2}{4} + 9$

$\frac{\pi^2}{4} + 9 = \left(\frac{\pi}{2}\right)^2 + \tan\left(\frac{\pi}{4}\right) + C$

$\therefore C = 8$

$\therefore y = x^2 + \tan\frac{x}{2} + 8$

5

$f(x) = \begin{cases} 2x^2 + ax + b & x \ge 1 \\ 3x - x^2 & x < 1 \end{cases}$

$\because$ The function is differentiable

$\therefore$ The function is continuous.

$\therefore f(1^-) = f(1^+) = f(1)$

$2 + a + b = 3 - 1 \qquad \therefore a = -b$

$\acute{f}(x) = \begin{cases} 4x + a & x \ge 1 \\ 3 - 2x & x < 1 \end{cases}$

$\because \acute{f}(1^-) = \acute{f}(1^+)$

$4 + a = 3 - 2 \qquad \therefore a = -3 \qquad \therefore b = 3$

$\acute{\acute{f}}(1^+) = \lim_{h \to 0^+} \frac{\acute{f}(h+1) - \acute{f}(1)}{h}$

$= \lim_{h \to 0^+} \frac{4(h+1) + a - (4 + a)}{h}$

$= \lim_{h \to 0^+} \frac{4h + 4 - 3 - 4 + 3}{h} = 4$

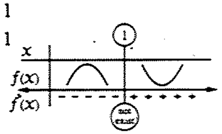
$\acute{\acute{f}}(1^-) = \lim_{h \to 0^-} \frac{\acute{f}(h+1) - \acute{f}(1)}{h}$

$= \lim_{h \to 0^-} \frac{3 - 2(h+1) - (4 + a)}{h}$

$= \lim_{h \to 0^-} \frac{3 - 2h - 2 - 4 + 3}{h} = -2$

$\acute{\acute{f}}(1)$ not exist.

$\acute{\acute{f}}(x) = \begin{cases} 4 & x > 1 \\ \text{not exist} & x = 1 \\ -2 & x < 1 \end{cases}$



The curve is convex upwards in $]-\infty , 1[$

and convex downwards in $]1 , \infty[$

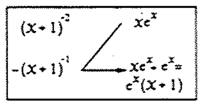
the function has tangent at $x = 1$

$\therefore$ has inflection point is (1 , 2)

6 ⓐ   7 ⓒ

8

[a] $\int \frac{x e^x}{(x+1)^2}\,dx$



$= -x(x+1)^{-1}e^x + \int e^x\,dx$

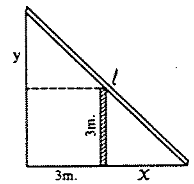
$= -x(x+1)^{-1}e^x + e^x + C$

[b] $\int \frac{2x}{\sqrt{x^2}}\,dx = 2\sqrt{x^2} + C$

9 ⓓ   10 ⓒ

11

$l = \sqrt{x^2 + 9} + \sqrt{y^2 + 9}$



From similarity $\frac{x}{3} = \frac{3}{y}$

$\therefore y = \frac{9}{x}$

$\therefore l = \sqrt{x^2 + 9} + \sqrt{\left(\frac{9}{x}\right)^2 + 9}$

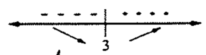
$= \sqrt{x^2 + 9}\,(1 + \frac{3}{x})$

$\acute{l} = \frac{-3}{x^2}\sqrt{x^2 + 9} + \frac{x}{\sqrt{x^2 + 9}} \times (1 + \frac{3}{x})$

$= \frac{-3(x^2 + 9) + x(x^2 + 3x)}{x^2\sqrt{x^2 + 9}}$

$= \frac{-3x^2 - 27 + x^3 + 3x^2}{x^2\sqrt{x^2 + 9}}$

$\acute{l} = \frac{x^3 - 27}{x^2\sqrt{x^2 + 9}} \qquad \therefore \text{at } \acute{l} = 0$



$\therefore x = 3$ minimum value

minimum value of length of ladder l

at $(x = 3) = \sqrt{9 + 9}\,(1 + \frac{3}{3}) = 6\sqrt{2}$ m

12

$\sin x = xy$ (by differentiation respect to x)

$\cos x = x\acute{y} + y$

$x\acute{y} = \cos x - y$ (by differentiation respect to x)

$x\acute{\acute{y}} + \acute{y} = -\sin x - \acute{y}$, $x\acute{\acute{y}} + 2\acute{y} + \sin x = 0 \times x$

$x^2\acute{\acute{y}} + 2x\acute{y} + x\sin x = 0$

$x^2\acute{\acute{y}} + 2x\acute{y} + x^2 y = 0$

$x^2(\acute{\acute{y}} + y) + 2(\cos x - y) = 0$

$x^2(\acute{\acute{y}} + y) + 2\cos x = 2y$

13 ⓒ

14

$\text{Area} = \int_0^4 y\,dx = \int_0^4 (3x + 4)^{\frac{1}{2}}\,dx$

$= \left[\frac{2}{9}(3x + 4)^{\frac{3}{2}}\right]_0^4$

$= \left[\frac{2}{9}(3(4) + 4)^{\frac{3}{2}} - \frac{2}{9}(4)^{\frac{3}{2}}\right]$

$= \frac{112}{9}$ square unit.

15 ⓓ   16 ⓐ

17

$g(x) = 2\ln x - x^2$

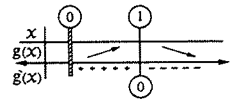
$\therefore$ Domain = $]0 , \infty[$

$\acute{g}(x) = \frac{2}{x} - 2x = \frac{2 - 2x^2}{x}$

Put $\acute{g}(x)$ = zero then $2 - 2x^2 = 0$

$\therefore x = 1$ or $x = -1 \notin$ Domain

$\acute{g}(x)$ undefined at $x = 0 \notin$ Domain



$\therefore$ Function is increasing in $]0 , 1[$

is decreasing in $]1 , \infty[$

18

Dimensions of cuboid at any instant

are 3 + 2 t , 4 + t , 12 − 3 t

$\therefore$ Volume of the cuboid at any instant

is $V = (3 + 2t)(4 + t)(12 - 3t)$

$= 144 + 96t - 9t^2 - 6t^3$

$\therefore \frac{dV}{dt} = 96 - 18t - 18t^2$

$\left(\frac{dV}{dt}\right)_{t=2} = -12$ cm³/sec.

If diagonal of cuboid = L (1st req.)

$\therefore L^2 = (3 + 2t)^2 + (4 + t)^2 + (12 - 3t)^2$

$2L\frac{dL}{dt} = 2(3 + 2t)2 + 2(4 + t) + 2(12 - 3t) \times -3$

(at t = 2 then L = 11)

$\therefore \frac{dL}{dt} = \frac{7 \times 2 + 6 + 6 \times -3}{11} = \frac{2}{11}$ (2nd req.)

Model 15

1 ⓐ

2

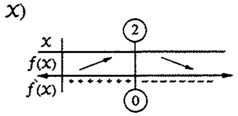
$f(x) = e^x(3 - x)$

$\acute{f}(x) = e^x \times -1 + e^x(3 - x)$

$= 2e^x - xe^x$

$= (2 - x)e^x$

$\acute{f}(x) = 0$ at $x = 2$



The function has local maximum value = $f(2) = e^2$

3 ⓑ   4 ⓒ   5 ⓑ

6 ⓓ   7 ⓑ   8 ⓐ

9

$\frac{dy}{dx} = a \csc^2 x$ (by integration with respect to x)

$y = -a \cot x + c$

The curve passes through the points $(\frac{\pi}{4}, 5), (\frac{3\pi}{4}, 1)$

$5 = -a + c$ (1)

$1 = a + c$ (2)

$\therefore c = 3 \;,\; a = -2$

$\therefore$ The equation $y = 2 \cot x + 3$

10

Let radius length of the sector $= x$

and length of the arc $= y$

$\therefore$ Its perimeter $= 2x + y = 30$

$\therefore y = 30 - 2x$

$\therefore$ Area (A) $= \frac{1}{2} x y$

$= \frac{1}{2} x (30 - 2x) = 15x - x^2$

$\acute{A} = 15 - 2x \,,\, \acute{\acute{A}} = -2$

$\acute{A} = 0$ at $x = 7.5$

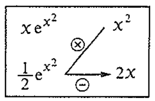
$\because (\acute{\acute{A}})_{(x=7.5)} = -2 < 0$ (Max.)

$\therefore$ Radius length $= 7.5$ cm.

11 ⓒ

12

[a] $\int x^3 e^{x^2} dx$



$= \int x^2 \cdot x e^{x^2} dx$

$= \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} \int 2x e^{x^2} dx$

$= \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + c$

[b] $\int \sec x^{2017} \tan x \, dx = \int \sec^{2016} x \sec x . \tan x$

$= \frac{1}{2017} \sec^{2017} x + c$

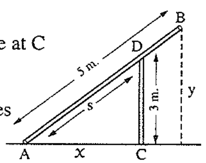
13

$\frac{dx}{dt} = \frac{5}{4}$ m./min.

$\because \Delta ACD$ right angle triangle at C

$\therefore AD = \sqrt{9 + x^2}$

From similarity of two triangles



$\therefore \frac{\sqrt{9+x^2}}{5} = \frac{3}{y}$

$\therefore y = \frac{15}{\sqrt{9+x^2}} = 15 (9 + x^2)^{\frac{-1}{2}}$

$\therefore \frac{dy}{dt} = 15 \times \frac{-1}{2} (9 + x^2)^{\frac{-3}{2}} \times 2x \times \frac{dx}{dt}$

When B at D then $x = 4$ cm.

$\therefore \frac{dy}{dt} = \frac{-3}{5} =$ m./min.

14

[a] $x = \sqrt{y}$ $\quad \therefore y_2 = x^2$

$x + y = 0$ $\quad y_3 = -x$

$x - y + 6 = 0$ $\quad \therefore y_1 = x + 6$

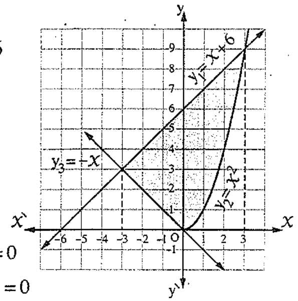
Point of intersection of two straight lines :

$y_1 = y_3$

$\therefore -x = x + 6$

$\therefore x = -3$

Point of intersection of y_2 , line y_1



$x^2 = x + 6$

$\therefore x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$\therefore x = 3$ or $x = -2$

In interval $[-3 , 0]$ $\quad y_1 > y_3$

In interval $[0 , 3]$ $\quad y_1 > y_2$

$\therefore$ area $= \int_{-3}^{0} (y_1 - y_3)\, dx + \int_{0}^{3} (y_1 - y_2)\, dx$

$= \int_{-3}^{0} (x + 6 + x)\, dx + \int_{0}^{3} (x + 6 - x^2)\, dx$

$= \int_{-3}^{0} (2x + 6)\, dx + \int_{0}^{3} (-x^2 + x + 6)\, dx$

$= [x^2 + 6x]_{-3}^{0} + [-\frac{1}{3} x^3 + \frac{1}{2} x^2 + 6x]_{0}^{3}$

$= 9 + \frac{27}{2} = \frac{45}{2}$ square unit.

[b] $y + x^2 = 6$, then

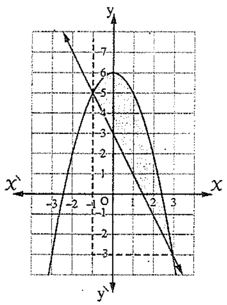
$y = -x^2 + 6 \longrightarrow (1)$

, $y + 2x - 3 = 0$

, then

$y = -2x + 3 \longrightarrow (2)$

By solving the two equations



$\therefore x = 3 , x = -1$

$\therefore$ The area

$= \int_{-1}^{3} [(-x^2 + 6) - (-2x + 3)]\, dx$

$= \int_{-1}^{3} (-x^2 + 2x + 3)\, dx$

$= [-\frac{1}{3} x^3 + x^2 + 3x]_{-1}^{3} = \frac{32}{3}$ square unit.

15

$y = e^{-x} \sqrt{\frac{1+x}{1-x}}$ by taking ln. to each sides

$\therefore \ln y = \ln e^{-x} + \frac{1}{2} \ln \frac{1+x}{1-x}$

(by differentiate with respect to x)

$\frac{1}{y} \times \grave{y} = -1 + \frac{1}{2} \times \frac{1-x}{1+x} \times \frac{(1-x)(1) - (1+x)(-1)}{(1-x)^2}$

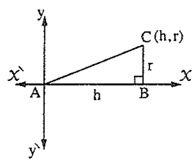
$\frac{\grave{y}}{y} = -1 + \frac{1}{1-x^2}$ by (multiply by $y (1 - x^2)$)

$\therefore (1 - x^2)\, \grave{y} = -y + x^2 y + y = x^2 y$

16 ⓐ $\quad$ **17** ⓓ

18

* We draw a right-angled triangle ABC where A is the origin point.

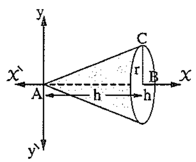


One of the sides of the right angle $\overline{AB}$ is coincided with the x-axis and the length of $\overline{AB} = h$

, the length of $\overline{BC} = r$

, $\because A (0 , 0) , C (h , r)$

$\therefore$ The equation of $\overleftrightarrow{AC}$ is $\frac{y-0}{x-0} = \frac{r-0}{h-0}$



i.e. $\boxed{y = \frac{r}{h} x}$

* The right circular cone is produced by the revolving of the triangle ABC bounded by the straight line $\overleftrightarrow{AC}$ and the x-axis and the two straight lines $x = 0 , x = h$

$\therefore$ The volume of the cone $= \pi \int_0^h y^2\, dx$

$= \pi \int_0^h \frac{r^2}{h^2} x^2\, dx = \frac{\pi r^2}{h^2} \int_0^h x^2\, dx$

$= \frac{\pi r^2}{h^2} [\frac{1}{3} x^3]_0^h = \frac{\pi r^2}{h^2} \times \frac{1}{3} h^3 = \frac{1}{3} \pi r^2 h$

Model 16

1 ⓐ $\quad$ **2** ⓐ $\quad$ **3** ⓑ $\quad$ **4** ⓓ

5

$x^2 - 2xy - y^2 = 1$ (by differentiate with respect to x)

$2x - 2x\grave{y} - 2y - 2y\grave{y} = 0$

$\grave{y} = \frac{x-y}{x+y}$ at $x = 1 , y = 0 : \grave{y} = 1$

$\therefore$ equation of tangent is :

$y - 0 = 1 (x - 1)$

$x - y - 1 = 0$

Slope of normal $= -1$

$\therefore$ equation of normal is $y - 0 = -1 (x - 1)$

$x + y - 1 = 0$

6 ⓒ

7

[a] $\int_3 \frac{dx}{\sqrt{2x+9}} = \int (2x + 9)^{-\frac{1}{3}} dx = \frac{3}{4} (2x + 9)^{\frac{2}{3}} + C$

[b] $\int [(1 - \cot x)^2 + 2 \cot x]\, dx$

$= \int (1 - 2 \cot x + \cot^2 x + 2 \cot x)\, dx$

$= \int (1 + \cot^2 x)\, dx = \int \csc^2 x\, dx = -\cot x + C$

8

$f(x) = \sin x$

$\therefore \acute{f}(x) = \cos x$

$\acute{f}(x) = 0$ $\qquad \therefore \cos x = 0$

, then $x = \frac{\pi}{2} \notin [\frac{5\pi}{6} , \frac{11\pi}{6}]$

or $x = \frac{3\pi}{2} \in [\frac{5\pi}{6} , \frac{11\pi}{6}]$

$\because f(\frac{5\pi}{6}) = \frac{1}{2} , f(\frac{3}{2}\pi) = -1$

, $f(\frac{11\pi}{6}) = \frac{-1}{2}$

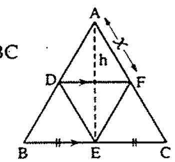
$\therefore$ The function has absolute maximum value $= \frac{1}{2}$ at $x = \frac{5\pi}{6}$

and has absolute minimum value $= -1$ at $x = \frac{11\pi}{6}$

9 ⓑ

10

From similarity of $\Delta\Delta ADF$, ABC



$\frac{x}{2\ell} = \frac{h}{\sqrt{3}\,\ell}$

$\therefore h = \frac{\sqrt{3}}{2}x$

$\therefore$ A (area of Δ DEF) $= \frac{1}{2}x \times \left(\sqrt{3}\,\ell - \frac{\sqrt{3}}{2}x\right)$

$= \frac{\sqrt{3}}{2}\ell x - \frac{\sqrt{3}}{4}x^2$

$\acute{A} = \frac{\sqrt{3}}{2}\ell - \frac{\sqrt{3}}{2}x$, put $\acute{A} = 0$, $\therefore x = \ell$

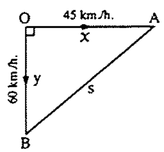
$\grave{A} = -\frac{\sqrt{3}}{2} < 0$ (Maximum value)

greatest area $= A_{x=\ell} = \frac{\sqrt{3}}{2}\ell(\ell) - \frac{\sqrt{3}}{4}\ell^2 = \frac{\sqrt{3}}{4}\ell^2$

$\therefore$ area of Δ ABC $= \frac{\sqrt{3}}{4}(2\ell)^2 = \sqrt{3}\,\ell^2$

$A = \frac{1}{4}$ area of Δ ABC

11



Let the 1^{st} train at a distance x km. from "O"
, the 2^{nd} trian at a distance y km. from "O"
and the distance between the two trains (S) km.

$\therefore S^2 = x^2 + y^2$

$\therefore 2S\frac{ds}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$ (1)

At 3 o'clok

$\therefore x = 4 \times 45 = 180$ km. , $y = 3 \times 60 = 180$ km.

$\therefore S = \sqrt{(180)^2 + (180)^2} = 180\sqrt{2}$ km.

From equation (1) : $2 \times 180\sqrt{2} \times \frac{ds}{dt}$

$= 2(180)(45) + 2(180)(60)$

$\therefore \frac{ds}{dt} = 52.5\sqrt{2}$ km./h.

12 ⓓ

13

$\because f'(x) = 6x^2 - 30x + 36$

$\therefore f'(x) = 6(x^2 - 5x + 6)$

$\therefore f'(x) = 6(x-2)(x-3)$

$\therefore f'(x) = 0$ when $x = 2$, $x = 3$

$\therefore f(x)$ has two critical points at $x = 2$, $x = 3$

$\because f''(x) = 12x - 30$

$\therefore f''(2) = 24 - 30 = -6 < 0$

$\therefore (2 , f(2))$ is point of local maximum value.

$\therefore f''(3) = 36 - 30 = 6 > 0$

$\therefore (3 , f(3))$ is point of local minimum value.

$\because$ Value of y at the point of maximum value = 28

$\therefore f(2) = 28$ $\therefore (2 , 28)$ on the curve.

$\because f'(x) = 6x^2 - 30x + 36$

$\therefore$ equation of family of curves is :

$f(x) = \int f'(x)\,dx$

i.e. $f(x) = \int (6x^2 - 30x + 36)\,dx$

i.e. $f(x) = 2x^3 - 15x^2 + 36x + c$

$\therefore f(2) = 28$

$\therefore 28 = 16 - 60 + 72 + c$

$\therefore c = 0$

$\therefore$ Equation of req.curve is $y = 2x^3 - 15x^2 + 36x$

14 ⓒ

15

[a] $y = e^{ax}$

$\therefore \frac{dy}{dx} = a\,e^{ax}$

$\therefore \frac{d^2y}{dx^2} = a^2 e^{ax}$

$\therefore \frac{d^3y}{dx^3} = a^3 e^{ax}$

$\therefore \frac{d^4y}{dx^4} = a^4 e^{ax} = a^4 y$

$\therefore \frac{d^4y}{dx^4} - a^4 y = 0$

[b] $\because \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

$\therefore a^2 e^{ax} - 2a\,e^{ax} + e^{ax} = 0$

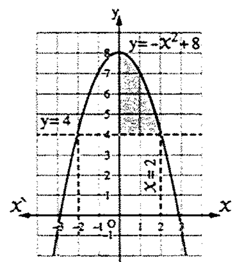
$e^{ax}(a^2 - 2a + 1) = 0$

$\therefore e^{ax}(a-1)^2 = 0$ $\therefore a = 1$

16 ⓓ **17** ⓑ

18

Point of intersection of the curve $y = -x^2 + 8$ and the line $y = 4$



$\therefore -x^2 + 8 = 4$

$\therefore x^2 = 4$

$x = 2$ or $x = -2$

volume $= \pi\int_4^8 x^2\,dy$

$= \pi\int_4^8 (-y + 8)\,dy$

$= \pi\left[-\frac{1}{2}y^2 + 8y\right]_4^8$

$= \pi\left(\left[-\frac{1}{2}(8)^2 + 8(8)\right] - \left[-\frac{1}{2}(4)^2 + 8(4)\right]\right)$

$= 8\pi$ cobic unit.

Model 17

1 ⓒ **2** ⓓ **3** ⓒ **4** ⓓ **5** ⓑ

6

$\because x^2 + y^2 = 5$ (1)

$\therefore 2x + 2y\frac{dy}{dx} =$ zero

$\frac{dy}{dx} = \frac{-x}{y}$

$\because \frac{dy}{dx} = \tan\theta$ $\therefore \frac{-x}{y} = 2$ $\therefore x = -2y$

By substite in the equation (1)

$\therefore (-2y)^2 + y^2 = 5$ $\therefore 4y^2 + y^2 = 5$

$\therefore y^2 = 1$ $\therefore y = 1$ or $y = -1$

, then $x = -2$ or $x = 2$

$\therefore$ Equation of the tangent at the point $(-2 , 1)$ is :

$y - 1 = 2(x + 2)$ $\therefore y = 2x + 5$

$\therefore$ Equation of the tangent at the point $(2 , -1)$ is :

$y + 1 = 2(x - 2)$ $\therefore y = 2x - 5$

7

[a] $\int x^5\left(1 + \frac{3}{x}\right)^5 dx = \int (x+3)^5\,dx = \frac{1}{6}(x+3)^6 + C$

[b] $\int \frac{1 + \sin^2 x}{1 - \sin^2 x}\,dx = \int \frac{1 + \sin^2 x}{\cos^2 x}\,dx$

$= \int (\sec^2 x + \tan^2 x)\,dx$

$= \int (2\sec^2 x - 1)\,dx$

$= 2\tan x - x + C$

8

$\frac{dy}{dx} = 3\cos 3x - 3\sin 3x$

$\frac{d^2y}{dx^2} = -9\sin 3x - 9\cos 3x$

$\frac{d^3y}{dx^3} = -27\cos 3x + 27\sin 3x$

$\frac{d^4y}{dx^4} = 81\sin 3x + 81\cos 3x$

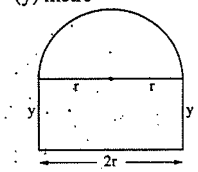
$= 81(\sin 3x + \cos 3x) = 81y$

9

Let the radius length of the circle = (r) metre

$\therefore$ The length of the rectangle = (2 r) metre

Let the width of the rectangle = (y) metre



, $\because$ The perimeter of the window = 6 m.

$\therefore 2y + 2r + \pi r = 6$

$\therefore y + r + \frac{1}{2}\pi r = 3.$

$\therefore y = 3 - r - \frac{1}{2}\pi r$ (1)

Let the area of the window = A

$\therefore A = 2ry + \frac{1}{2}\pi r^2$ (2)

By substituting from (1) in (2) :

$\therefore A = 2r\left(3 - r - \frac{1}{2}\pi r\right) + \frac{1}{2}\pi r^2$

$= 6r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2 = 6r - 2r^2 - \frac{1}{2}\pi r^2$

$\therefore \frac{dA}{dr} = 6 - 4r - \pi r$

, at the maximum or minimum values , put $\frac{dA}{dr} = 0$

$\therefore 6 - 4r - \pi r = 0$ $\therefore r = \frac{6}{4+\pi}$ m.

, $\because \frac{d^2A}{dr^2} = -4 - \pi < 0$

$\therefore$ at $r = \frac{6}{4+\pi}$

, the area of the window (A) is maximum.

10 ⓒ **11** ⓐ

12

Let the equation of the curve is :

$y = a x^3 + b x^2 + c x + d$

When a , b , c , d are real constant.

$\therefore y' = 3a x^2 + 2b x + c$

$y'' = 6a x + 2b$

$\because f''(x) < 0$ when $x < \frac{-2}{3}$

$f''(x) > 0$ when $x > \frac{-2}{3}$

$\therefore$ At $x = \frac{-2}{3}$ has point of inflection

$\dot{\dot{y}} = 0$, at $x = \frac{-2}{3}$

$\therefore -4a + 2b = 0$

$\therefore b = 2a$ (1)

$\because (-1, 2)$ is critical point.

$\therefore \dot{y} = 0$, when $x = -1$

$\therefore 3a - 2b + c = 0$ (2)

$\because$ The curve passes through the point (1 , 6)

$\therefore a + b + c + d = 6$ (3)

$\because$ The curve passes through (− 1 , 2)

$-a + b - c + d = 2$ (4)

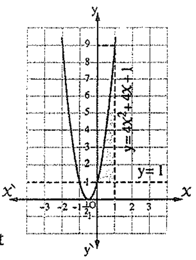
By solving the 4 equations , we get :

$\therefore a = 1 , b = 2 , c = 1 , d = 2$

$\therefore$ equation of the curve is :

$y = x^3 + 2x^2 + x + 2$

13



$\text{Area} = {}_0\int^1 (y_2 - y_1)\,dx$

$= {}_0\int^1 [(4x^2 + 4x + 1) - (1)]\,dx$

$= {}_0\int^1 (4x^2 + 4x)\,dx$

$= [\frac{4}{3}x^3 + 2x^2]_0^1$

$= \left(\frac{10}{3}\right) - (0) = \frac{10}{3}$ square unit

14 ⓑ **15** ⓒ **16** ⓐ

17

$\because \frac{dy}{dx} = \frac{\sqrt{2y+1}}{\sqrt{3x-2}}$

$\therefore \frac{dy}{\sqrt{2y+1}} = \frac{dx}{\sqrt{3x-2}}$

$\therefore \int (2y+1)^{-\frac{1}{2}}\,dy = \int (3x-2)^{-\frac{1}{2}}\,dx$

$\therefore (2y+1)^{\frac{1}{2}} = \frac{2}{3}(3x-2)^{\frac{1}{2}} + c$

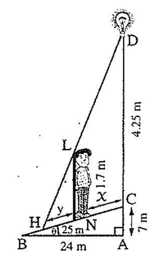
$\therefore$ the curve passes through the point (1 , 4)

$\therefore 3 = \frac{2}{3} + c \Rightarrow c = \frac{7}{3}$

Equation of the curve is :

$\sqrt{2y+1} = \frac{2}{3}\sqrt{3x-2} + \frac{7}{3}$

18



1^{st} In Δ ABC :

$\because \tan\theta = \frac{7}{24}$

$\therefore AC = 7k$

$, AB = 24k$

$\because CB = 25$ m.

$\therefore (7k)^2 + (24k)^2 = (25)^2$

$\therefore k = 1$

$\therefore AC = 7$ m. , $AB = 24$ m.

$\therefore CD = 11\frac{1}{4} - 7 = 4\frac{1}{4}$ m.

$, \because \Delta HCD \sim \Delta HNL$

$\therefore \frac{1.7}{4.25} = \frac{y}{x+y}$ $\qquad \therefore \frac{2}{5} = \frac{y}{x+y}$

$\therefore 2x = 3y$ $\qquad \therefore 2\frac{dx}{dt} = 3\frac{dy}{dt}$

$\therefore 2(-6) = 3\left(\frac{dy}{dt}\right)$ $\qquad \therefore \frac{dy}{dt} = -4$ m./min.

$\therefore$ The rate of decreasing of the length of the man's shadow = 4 m./min.

2^{nd} : $S = x + y$

$\therefore \frac{ds}{dt} = \frac{dx}{dt} + \frac{dy}{dt} = -6 - 4 = -10$ m./min.

$\therefore$ The end of the man's shadow approaches by rate 10 m./min. from the upper point of the inclined plane.

Model 18

1 ⓒ **2** ⓓ **3** ⓑ **4** ⓐ

5

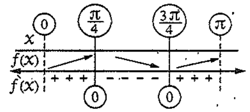
[a] $\because f(x) = \sin x \cos x + 5$

$\therefore f(x) = \frac{1}{2}\sin 2x + 5$

$\therefore \dot{f}(x) = \cos 2x$ $\qquad \therefore \dot{f}(x) = 0$

, when $2x = \frac{\pi}{2}$ or $2x = \frac{3\pi}{2}$

$\therefore x = \frac{\pi}{4}$ or $x = \frac{3\pi}{4}$



$\therefore$ has local maximum value at $x = \frac{\pi}{4}$ and its value $= 5\frac{1}{2}$

has local minimum value at $x = \frac{3\pi}{4}$ and its value $= 4\frac{1}{2}$

[b] $\because f(x) = -3x^5 + 5x^3$

$\therefore \dot{f}(x) = -15x^4 + 15x^2$

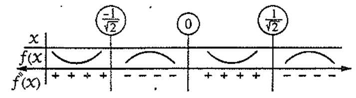
$\therefore \ddot{f}(x) = -60x^3 + 30x$

$\therefore \dddot{f}(x) = 0$ then $-60x^3 + 30x = 0$

$\therefore 30x(-2x^2 + 1) = 0$

$\therefore x = 0$ or $2x^2 - 1 = 0$

$\therefore x^2 = \frac{1}{2}$ $\qquad \therefore x = \pm\frac{1}{\sqrt{2}}$



$\therefore$ The curve is convex upwards in each of

$]\frac{-1}{\sqrt{2}}, 0[$, $]\frac{1}{\sqrt{2}}, \infty[$

and the curve is convex downwards in each of

$]-\infty, \frac{-1}{\sqrt{2}}[$, $]0, \frac{1}{\sqrt{2}}[$

and inflection points are $\left(\frac{-1}{\sqrt{2}}, \frac{-7\sqrt{2}}{8}\right)$

, (0 , 0) , $\left(\frac{1}{\sqrt{2}}, \frac{7\sqrt{2}}{8}\right)$

6 ⓑ

7

$4y^3 = 3x^2$ (by differentiate respect to x)

$\therefore 12y^2\frac{dy}{dx} = 6x$ (divided by 6)

$\therefore 2y^2\frac{dy}{dx} = x$

(by differentiate respect to x)

$\therefore 2\left[y^2\frac{d^2y}{dx^2} + \frac{dy}{dx} \times 2y\frac{dy}{dx}\right] = 1$

$2y^2\frac{d^2y}{dx^2} + 4y\left(\frac{dy}{dx}\right)^2 = 1$

8 ⓑ **9** ⓒ

10

Let the length of sides of the triangle are x , y and z

$\therefore x + y + z = 40$ (by differentiate respect to time)

$\therefore \frac{dx}{dt} + \frac{dy}{dt} + \frac{dz}{dt} = 0$

$\because \frac{dz}{dt} = 7$ $\qquad \therefore \frac{dx}{dt} + \frac{dy}{dt} = -7$ (1)

$\therefore x^2 + y^2 = z^2$ (by differentiate respect to time)

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$x = 8$, $y = 15$, $z = 17$, $\frac{dz}{dt} = 7$

$, 2 \times 8 \times \frac{dz}{dt} + 2 \times 15 \times \frac{dy}{dt} = 2 \times 17 \times 7$

$\therefore 8\frac{dx}{dt} + 15\frac{dy}{dt} = 119$ (2)

by solving the two equations (1) , (2)

we get $\frac{dx}{dt} = -32$ cm./min. , $\frac{dy}{dt} = 25$ cm./min.

11 ⓐ

12

$\frac{d^2y}{dx^2} = 2x^{-3}$ (by integration respect to x)

$\therefore \frac{dy}{dx} = -x^{-2} + c_1$

$\therefore$ Slope of the tangent at the point $(2, \frac{5}{2})$ is $\frac{3}{4}$

$\therefore \frac{3}{4} = -(2)^{-2} + c_1$ $\qquad \therefore c_1 = 1$

$\frac{dy}{dx} = -x^{-2} + 1$ (by integration respect to x)

$\therefore y = x^{-1} + x + c_2$ by substitute by the point $\left(2, \frac{5}{2}\right)$

$\therefore \frac{5}{2} = 2^{-1} + (2) + c_2$ $\qquad \therefore c_2 = 0$

$\therefore y = x^{-1} + x$

13 ⓒ

14

[a] $\int \frac{x+2}{x^2+4x+3}\,dx = \frac{1}{2}\int \frac{2x+4}{x^2+4x+3}\,dx$

$= \frac{1}{2}\ln|x^2 + 4x + 3| + C$

[b] $\int \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}\,dx$

$= \int \frac{(\sin x + \cos x)(\sin^2 x - \sin x \cos x + \cos^2 x)}{(\sin x + \cos x)}\,dx$

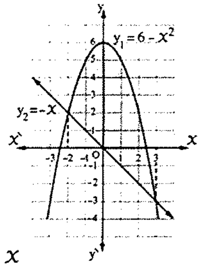
$= \int \left(1 - \frac{1}{2}\sin 2x\right)dx = x + \frac{1}{4}\cos 2x + C$

15 ⓐ

16

Equation of straight line passes through two points $(3 , -3) , (-2 , 2)$ is $y = -x$

to find points of intersection of the curve y_1 and the straight line y_2



$y_1 = 6 - x^2 , y_2 = -x$

$\therefore 6 - x^2 = -x$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$

$x = -2$

$\therefore$ The area $= {}_{-2}\int^{3} (y_1 - y_2)\, dx$

where $y_1 > y_2$ in the interval $[-2 , 3]$

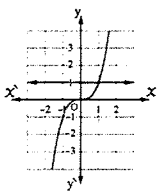
$= {}_{-2}\int^{3} \left((6-x^2)-(-x)\right) dx = {}_{-2}\int^{3} (-x^2 + x + 6)\, dx$

$= [-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 6x]_{-2}^{3}$

$= [-\frac{1}{3}(3)^3 + \frac{1}{2}(3)^2 + 6(3)] - [-\frac{1}{3}(-2)^3 + \frac{1}{2}(-2)^2 + 6(-2)]$

$= \frac{125}{6}$

17



$y_1 = 1 , y_2 = x^3$

$\because y_1 \geq y_2$ in interval $[0 , 1]$

$\therefore$ The volume

$= \pi\, {}_0\int^1 (y_1^2 - y_2^2)\, dx$

$= \pi\, {}_0\int^1 (1 - x^6)\, dx$

$= \pi\, [x - \frac{1}{7}x^7]_0^1 = \frac{6}{7}\pi$ cubic unit

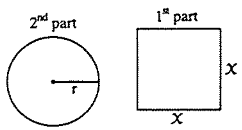
$\therefore \pi r^2 \times 42 = \frac{6}{7}\pi$

$\therefore r = \frac{1}{7}$ length unit

18

Let the length of side of the square $= x$

and length of radius $= r$



$\because$ Perimeter of the two shapes

$= 4x + 2\pi r = 34$

$\therefore x = \frac{17 - \pi r}{2} = \left(\frac{17}{2} - \frac{\pi}{2} r\right)$ cm.

$\therefore$ Sum of areas of the two shapes $= x^2 + \pi r^2$

$A = \left(\frac{17}{2} - \frac{\pi}{2} r\right)^2 + \pi r^2$

$\therefore \frac{dA}{dr} = 2\left(\frac{17}{2} - \frac{\pi}{2} r\right)\left(\frac{-\pi}{2}\right) + 2\pi r$

$= \frac{-17}{2}\pi + \frac{\pi^2 r}{2} + 2\pi r$ at $\frac{dA}{dr} = 0$

$\frac{-17}{2} + \frac{\pi r}{2} + 2r = 0$

$\therefore r.\left(\frac{\pi}{2} + 2\right) = \frac{17}{2}$

$\therefore r(\pi + 4) = 17$ $\qquad \therefore r = \frac{17}{\pi + 4}$

$\frac{d^2 A}{dr^2} = \frac{\pi^2}{2} + 2\pi > 0$

$\therefore$ The sum of the two areas is minimum when $r = \frac{17}{\pi + 4}$

$\therefore$ Length of the 2nd part $= 2\pi r = \frac{34\pi}{\pi + 4}$

Length of the 1st part $= 34 - \frac{34\pi}{\pi + 4} = \frac{136}{\pi + 4}$

Model 19

1 ⓒ **2** ⓐ **3** ⓐ **4** ⓓ

5

Equation of the straight line $y_1 = -x + 3$

Equation of the curve : $x^2 = 4y$

$\therefore y_2 = \frac{1}{4}x^2$

$\because$ Equation of the curve

$y^2 = 4x$

$\therefore y_3 = 2\sqrt{x}$ (where y lies in the 1st quad.)

Points of intersection between the straight line (y_1) and the curve (y_2)

$\therefore -x + 3 = \frac{1}{4}x^2 \qquad \therefore x^2 + 4x - 12 = 0$

$\therefore x = -6 \notin$ 1st quad. $\quad , x = 2 \in$ 1st quad.

Points of intersection between straight line (y_1) and the curve (y_3)

$\therefore -x + 3 = 2\sqrt{x}$

$x + 2\sqrt{x} - 3 = 0 \quad , \therefore \left(\sqrt{x} + 3\right)\left(\sqrt{x} - 1\right) = 0$

$\therefore \sqrt{x} = -3$ (refused)

or $x = 1$

$\therefore y = -1 + 3 = 2 \in$ 1st quad.

$\therefore$ The area $= {}_0\int^1 (y_3 - y_2)\, dx + {}_1\int^2 (y_1 - y_2)\, dx$

$= {}_0\int^1 \left(2x^{\frac{1}{2}} - \frac{1}{4}x^2\right) dx$

$+ {}_1\int^2 (-x + 3) - \left(\frac{1}{4}x^2\right) dx$

$= [\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{12}x^3]_0^1 + [-\frac{1}{12}x^3 - \frac{1}{2}x^2 + 3x]_1^2$

$= \left(\frac{4}{3} - \frac{1}{12}\right) - (\text{zero}) + \left(\frac{-2}{3} - 2 + 6\right)$

$- \left(\frac{-1}{12} - \frac{1}{2} + 3\right) = \frac{13}{6}$

6

$\because y = x^2$, and let the point of tangency is (a , b)

$\therefore b = a^2$ $\qquad$ (1)

$\therefore \frac{dy}{dx} = 2x$, at $x = a$, then $\frac{dy}{dx} = 2a$

$\therefore$ Equation of tangent is

$y - y_1 = m(x - x_1) \qquad y - b = 2a(x - a)$

$y - a^2 = 2a(x - a)$

$\because$ The point $(3 , 5)$ lies on the tangent

$\therefore 5 - a^2 = 2a(3 - a) \qquad \therefore 5 - a^2 = 6a - 2a^2$

$\therefore a^2 - 6a + 5 = 0 \qquad \therefore (a-1)(a-5) = 0$

$a = 1$ or $a = 5$, then $b = 1$ or $b = 25$

$\therefore$ Equation of the tangent is $(y - 1) = 2(x - 1)$

i.e. $y - 2x + 1 = 0$ or $y - 25 = 10(x - 5)$

i.e. $y - 10x + 25 = 0$

7 ⓒ **8** ⓑ

9

$\because y = x^{-1} \qquad \therefore \frac{dy}{dx} = -x^{-2}$

$\therefore \frac{d^2 y}{dx^2} = 2x^{-3}$

$\therefore$ R.H.S $= x^2 \frac{d^2 y}{dx^2} + 3x\frac{dy}{dx} + y$

$= x^2(2x^{-3}) + 3x(-x^{-2}) + x^{-1}$

$= 2x^{-1} - 3x^{-1} + x^{-1} = \text{zero} = $ L.H.S

10

$f(x) = \begin{cases} x^2 - 2x & x \geq 2 \\ -x^2 + 2x & x < 2 \end{cases}$

$\hat{f}(2^+) = \lim_{h \to 0^+} \frac{(2+h)^2 - 2(2+h) - 0}{h} = 2$

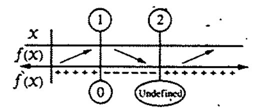
$\hat{f}(2^-) = \lim_{h \to 0^-} \frac{-(2+h)^2 + 2(2+h) - 0}{h} = -2$

$\therefore \hat{f}(2^+) \neq \hat{f}(2^-)$

$\therefore$ The function is not differentiable at $x = 2$

$\therefore \hat{f}(x) = \begin{cases} 2x - 2 & x > 2 \\ \text{not exist} & x = 2 \\ -2x + 2 & x < 2 \end{cases}$

$\therefore \hat{f}(x) = 0$ when $x = 1$ and is unfined at $x = 2$



$\therefore$ The function is increasing in $]-\infty , 1[,]2 , \infty[$

decreasing in $]1 , 2[$ $\qquad$ (1st req.)

$f(\frac{1}{2}) = \frac{3}{4} \quad , \quad f(1) = 1$

$f(2) = \text{zero} \quad , \quad f\left(\frac{5}{2}\right) = \frac{5}{4}$

$\therefore$ The absolute maximum value $= \frac{5}{4}$

, the absolute minimum value $=$ zero $\qquad$ (2nd req.)

11 ⓐ **12** ⓓ **13** ⓑ

14

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (Mult.by $a^2 b^2$)

$\therefore b^2 x^2 + a^2 y^2 = a^2 b^2$

$\therefore y^2 = \frac{a^2 b^2 - b^2 x^2}{a^2}$

When $y = 0$, then $x = \pm a$

$\therefore$ The volume $= \pi\, {}_{-a}\int^{a} y^2\, dx$

$= \pi\, {}_{-a}\int^{a} \left(b^2 - \frac{b^2 x^2}{a^2}\right) dx = 2\pi\, {}_0\int^{a} \left(b^2 - \frac{b^2 x^2}{a^2}\right) dx$

$= 2\pi\, [b^2 x - \frac{1}{3}\frac{b^2 x^3}{a^2}]_0^a$

$= 2\pi\left(a b^2 - \frac{1}{3} a b^2\right) = \frac{4}{3} a b^2 \pi$

15 ⓑ

16

[a] $\int (\csc x + \sin x)^2\, dx$

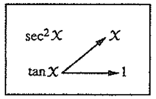
$= \int (\csc^2 x + 2\csc x . \sin x + \sin^2 x)\, dx$

$= \int \left(\csc^2 x + 2 + \frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$

$= \int \left(\csc^2 x - \frac{1}{2}\csc^2 x + 2\,\frac{1}{2}\right) dx$

$= -\cot x - \frac{1}{4}\sin 2x + 2\,\frac{1}{2}\,x + C$

[b] $\int x \sec^2 x$



$= x \tan x - \int \tan x \, dx$

$= x \tan x - \ln|\sec x| + C$

17

Let the points on the form (x , y) , distant "S"

$S = \sqrt{(x-0)^2 + (y-2)^2} = \sqrt{x^2 + y^2 - 4y + 4}$ (1)

$\because x^2 = y^2 + 8$ by substitute in (1)

$\therefore S = \sqrt{y^2 + 8 + y^2 - 4y + 4} = \sqrt{2y^2 - 4y + 12}$

$\therefore \frac{dS}{dy} = \frac{4y-4}{2\sqrt{2y^2-4y+12}} = \frac{2y-2}{\sqrt{2y^2-4y+12}}$

$\frac{dS}{dy}$ = zero when y = 1

$\because$ sign of $\frac{dS}{dy}$ changes before and after y = 1

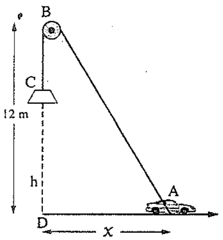
$\therefore$ at y = 1 has L. minimum value

$x^2 = 8 + 1 = 9$, $x = \pm 3$

the points are (3 , 1) , (– 3 , 1)

18

Let the height of the mass = h and the distance of the car from the projection of the pulley $= x$



$\therefore BC = 12 - h$

, $AB = 25 - (12 - h)$

$= 13 + h$

From the figure :

$\therefore (13 + h)^2 = x^2 + 12^2$ (1)

$\therefore 2(13 + h)\frac{dh}{dt} = 2x\frac{dx}{dt}$

At x = 16 m. , from equation (1) :

$\therefore (13 + h)^2 = 400$ $\therefore$ h = 7 m.

$\therefore 2(13 + 7)\frac{dh}{dt} = 2 \times 16 \times 6$

$\therefore \frac{dh}{dt} = 4.8$ m./sec.

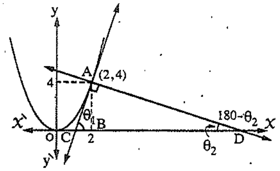
Model 20

1 ⓒ **2** ⓒ **3** ⓓ **4** ⓑ

5

$\acute{f}(x) = 2x$, at the point (2 , 4)

$\therefore \acute{f}(2) = 4$



$\because$ Slope of the tangent = $\tan\theta_1$

$\therefore \tan\theta_1 = \frac{AB}{BC}$

$\therefore 4 = \frac{4}{BC}$

$\therefore$ BC = 1 length unit

$\because$ Slope of the normal $= \frac{-1}{4}$

$\because \tan(180° - \theta_2) = -\tan\theta_2 = \frac{-1}{4} \Rightarrow \tan\theta_2 = \frac{1}{4}$

, $\because \tan\theta_2 = \frac{AB}{DB}$ $\therefore \frac{4}{BD} = \frac{1}{4}$

$\therefore$ DB = 16 length units

$\therefore$ DC = 17 length unit

$\therefore$ Area of Δ ABD $= \frac{1}{2}$ DC × AB $= \frac{1}{2} \times 17 \times 4$

= 34 square unit

6

$\because y = x \tan x$

$\therefore \frac{dy}{dx} = x\sec^2 x + \tan x$

By differentiation with respect to x

$\therefore \frac{d^2y}{dx^2} = \sec^2 x + 2x\tan x \sec^2 x + \sec^2 x$

$= 2\sec^2 x + 2x\tan x\sec^2 x$

$= 2\sec^2 x(1 + x\tan x)$

$\therefore \frac{d^2y}{dx^2} = 2\sec^2 x(1 + y)$

7

[a] $\int (a^{3\log_a x} + a^x + \cos\theta)\,dx$

$= \int (a^{\log_a x^3} + a^x + \cos\theta)\,dx$

$= \int (x^3 + a^x + \cos\theta)\,dx$

$= \frac{1}{4}x^4 + \frac{a^x}{\ln a} + x\cos\theta + c$

[b] $\int \left(8\sin^2\frac{x}{2}\cos^2\frac{x}{2}\right) dx$

$= \int 2\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2$

$= \int 2\sin^2 x\,dx = \int (1 - \cos 2x)\,dx$

$= x - \frac{1}{2}\sin 2x + c$

8 ⓑ **9** ⓓ **10** ⓐ

11

$\frac{dv}{dt} = 0.4t - 40$

$\therefore v = 0.2t^2 - 40t + c$

$\because$ t = 30 when v = 980

$\therefore 980 = 0.2(30)^2 - 40(30) + c$

$\therefore c = 2000$

$\therefore v = 0.2t^2 - 40t + 2000$

when t = 0 we get its capacity which is 2000 cm³ and it becomes empty when v = 0

$\therefore 0.2t^2 - 40t + 2000 = 0$

$\therefore t^2 - 200t + 10000 = 0$

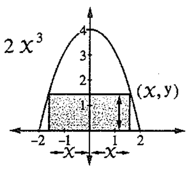
$\therefore (t - 100)^2 = 0$ $\therefore$ t = 100 sec.

12 ⓓ **13** ⓒ

14

The area of rectangle (A)

$= 2xy = 2x(4 - x^2) = 8x - 2x^3$



$\therefore \frac{dA}{dx} = 8 - 6x^2$

, $\because \frac{dA}{dx} = 0$

$\therefore x = \frac{2\sqrt{3}}{3}$, (negative refused)

In this case $y = \frac{8}{3}$

, $\because \frac{d^2A}{dx^2} = -12x < 0$

$\therefore$ There is a local maximum value.

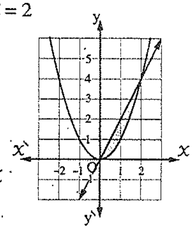
$\therefore$ The dimensions of the rectangle are : $\frac{4\sqrt{3}}{3}$, $\frac{8}{3}$

15 ⓐ

16

[a] $y_1 = x^2$, $y_2 = 2x$

Intersecting at $x = 0$ and $x = 2$



$\because y_2 \ge y_1$ in [0 , 2]

$\therefore$ The volume

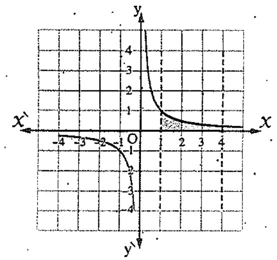
$= \pi \int_0^2 (y_2^2 - y_1^2)\,dx$

$= \pi \int_0^2 \left((2x)^2 - (x^2)^2\right) dx$

$= \pi \int_0^2 (4x^2 - x^4)\,dx$

$= \pi \left[\frac{4}{3}x^3 - \frac{1}{5}x^5\right]_0^2 = \frac{64}{15}\pi$ cubic unit

[b]



The volume $= \pi \int_1^4 y^2\,dx = \pi \int_1^4 \left(\frac{1}{x}\right)^2 dx$

$= \pi \int_1^4 x^{-2}\,dx = \pi\left[-x^{-1}\right]_1^4$

$= \frac{3}{4}\pi$ cubic unit

17

Length of sides of right angle after t min are $(8 - \frac{1}{2}t)$ cm , $(6 + t)$ cm

$\therefore$ Area of the triangle $= \frac{1}{2}(8 - \frac{1}{2}t)(6 + t)$

$\therefore A = 24 + \frac{5}{2}t - \frac{1}{4}t^2$

$\therefore \frac{dA}{dt} = \frac{5}{2} - \frac{1}{2}t$

$\therefore \left[\frac{dA}{dt}\right]_{t=2} = \frac{5}{2} - \frac{1}{2}(2) = 1.5$ cm²/min.

When $\frac{dA}{dt} = 0$

$\therefore \frac{5}{2} - \frac{1}{2}t = 0$ $\therefore$ t = 5 min

18

$y = ax^3 + bx^2 + cx + d$

$\therefore \frac{dy}{dx} = 3ax^2 + 2bx + c$

$\because$ (0 , 8) is a point of local maximum value

$\therefore \frac{dy}{dx} = 0$ at $x = 0 \quad \therefore c = 0$

$\because$ (1 , 7) is point of local minimum value

$\therefore \frac{dy}{dx} = 0$ at $x = 1$

$\therefore$ zero = 3 a + 2 b (1)

$\because$ (0 , 8) , (1 , 7) lying on the curve

$\therefore d = 8 , 7 = a + b + c + d \quad , \quad 7 = a + b + 8$

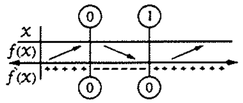
$\therefore a + b = -1$ (2)

From (1) , (2) we get a = 2 , b = − 3

$\therefore y = 2x^3 - 3x^2 + 8$

$\therefore y' = 6x^2 - 6x , y' = 0$

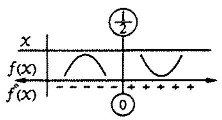
$\therefore 6x(x-1) = 0 , x = 0$ or $x = 1$



$\therefore$ The function of the curve is increasing in $]1 , \infty[$, $]-\infty , 0[$ and decreasing in $]0 , 1[$

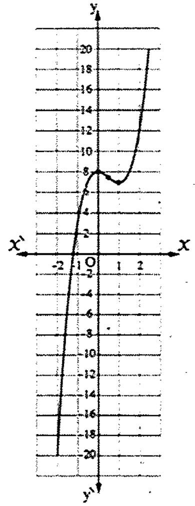
$f''(x) = 12x - 6 , f''(x) = 0$

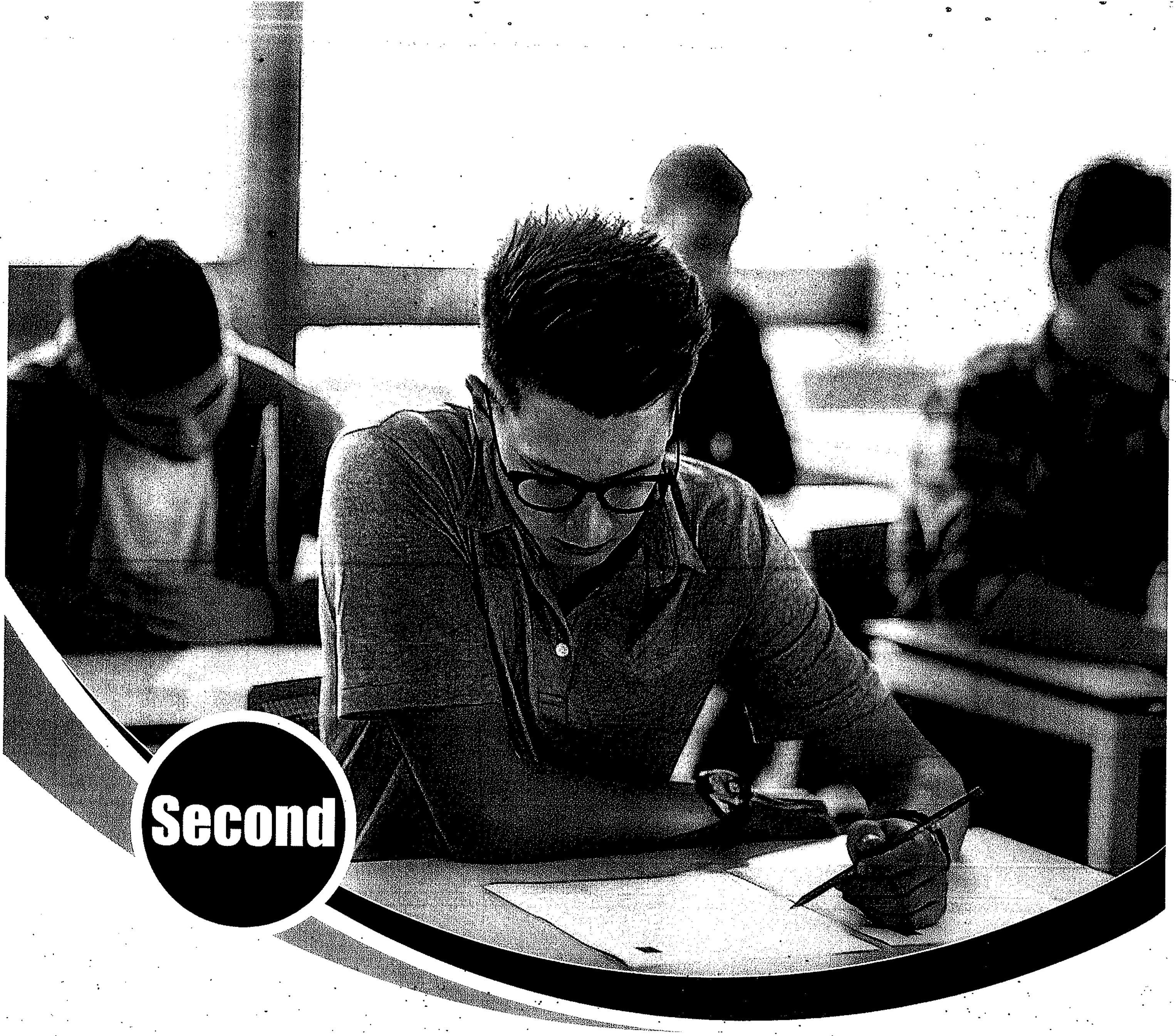
$\therefore 12x - 6 = 0 \quad \therefore x = \frac{1}{2}$



$\therefore$ The curve is convex downward in $]\frac{1}{2} , \infty[$ and convex upward in $]-\infty , \frac{1}{2}[$

$\therefore$ It has inflection point at $x = \frac{1}{2}$, and it is $\left(\frac{1}{2} , \frac{15}{2}\right)$





Guide Answers of

School Book Examinations

Model 1

1

(1) (c) (2) (b) (3) (b)
(4) (b) (5) (d) (6) (c)

2

[a] $\int \sin x \cos^3 x \, dx = -\int -\sin x \cos^3 x \, dx$

$= -\frac{1}{4}\cos^4 x + c$

[b] $e^{xy} - x^2 + y^3 = 0$ (1)

at $x = 0$ $\therefore 1 - 0 + y^3 = 0$

$\therefore y^3 = -1$ $\therefore y = -1$

(By differentiating (1) with respect to x)

$\therefore e^{xy}(y + x\grave{y}) - 2x + 3y^2 \grave{y} = 0$, at $(0, -1)$

$\therefore 1(-1+0) - 0 + 3(-1)^2 \grave{y} = 0$

$\therefore -1 + 3\grave{y} = 0$ $\therefore \grave{y} = \frac{1}{3}$

$\therefore \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{3}$

3

[a] $x^2 - 3xy - y^2 + 3 = 0$

(By differentiating with respect to x)

$\therefore 2x - 3y - 3x\frac{dy}{dx} - 2y\frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx} = \frac{2x - 3y}{3x + 2y}$ $\therefore \left(\frac{dy}{dx}\right)_{(-1,4)} = \frac{-14}{5}$

$\therefore$ The equation of the tangent is : $\frac{y-4}{x+1} = \frac{-14}{5}$

$\therefore 14x + 5y - 6 = 0$

[b] Let the two right sides after t minutes are :

$\left(6 + \frac{1}{3}t\right)$, $(30 - t)$

(1) Area of triangle A $= \frac{1}{2}\left(6 + \frac{1}{3}t\right) \times (30 - t)$

$= \frac{1}{2}\left(180 + 4t - \frac{1}{3}t^2\right)$

$= 90 + 2t - \frac{1}{6}t^2$

$\therefore \frac{dA}{dt} = 2 - \frac{1}{3}t$

at t = 3 : $\frac{dA}{dt} = 2 - \frac{1}{3}(3) = 1$ cm²/min.

(2) At $\frac{dA}{dt} = 0$ $\therefore 2 - \frac{1}{3}t = 0$

$\therefore t = 6$

4

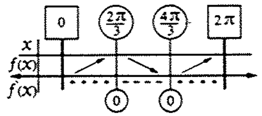
[a] $f(x) = x + 2\sin x$ $\because$ The domain $=]0, 2\pi[$

, $\grave{f}(x) = 1 + 2\cos x$

, putting : $\grave{f}(x) = 0$ $\therefore 1 + 2\cos x = 0$

$\therefore \cos x = \frac{-1}{2}$ $\therefore x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

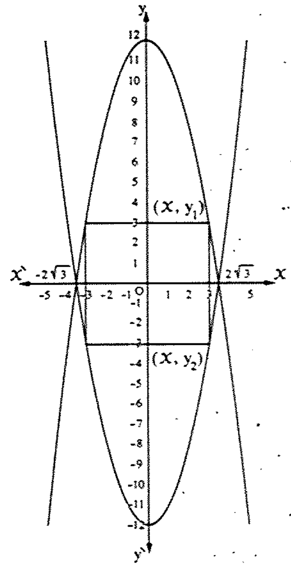
$\therefore$ The critical point is at $x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$



$\therefore f$ is increasing on $]0, \frac{2\pi}{3}[$, $]\frac{4\pi}{3}, 2\pi[$

and decreasing on $]\frac{2\pi}{3}, \frac{4\pi}{3}[$

[b]



$y_1 = 12 - x^2$ $0 < x < 2\sqrt{3}$

$y_2 = x^2 - 12$ $0 < x < 2\sqrt{3}$

$\therefore$ The dimensions of rectangle $= 2x$ horizontal

, vertical $= y_1 - y_2 = (12 - x^2) - (x^2 - 12)$

$= 12 - x^2 + 12 - x^2 = 24 - 2x^2$

$\therefore$ Area of rectangle (A) $= 2x(24 - 2x^2)$

$= 48x - 4x^3$

$\therefore \grave{A} = 48 - 12x^2$

Putting : $\grave{A} = 0$ $\therefore x^2 = 4$ $\therefore x = 2$

, $\grave{\grave{A}} = -24x$ $\therefore [\grave{\grave{A}}]_{x=2} < 0$

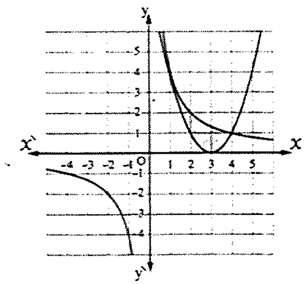
$\therefore$ at $x = 2$ A has a maximum value.

$\therefore$ The greatest area of rectangle

$A = 48 \times 2 - 4 \times 2^3 = 64$

5

[a]



$y_1 = \frac{4}{x}$, $y_2 = (x - 3)^2$

They intersect at $x = 1$, $x = 4$

, $\because y_1 \geq y_2$ in this interval

$\therefore$ The volume $= \pi \int_1^4 (y_1^2 - y_2^2)\, dx$

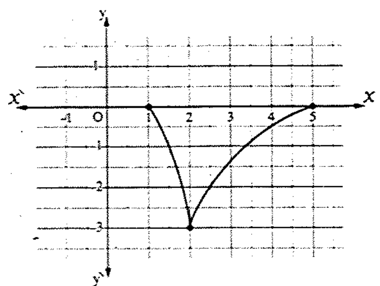
$= \pi \int_1^4 \left[\left(\frac{4}{x}\right)^2 - (x-3)^4\right] dx$

$= \pi \int_1^4 \left[\frac{16}{x^2} - (x-3)^4\right] dx$

$= \pi \left[-16x^{-1} - \frac{1}{5}(x-3)^5\right]_1^4$

$= 5.4\,\pi$ cubic unit.

[b]



Model 2

1

(1) (b) (2) (c) (3) (b)
(4) (a) (5) (c) (6) (c)

2

[a] (1) $\int x(2x-1)^3\, dx$

Putting $z = 2x - 1$

$\therefore x = \frac{1}{2}z + \frac{1}{2}$ $\therefore dx = \frac{1}{2}dz$

$\therefore \int x(2x-1)^3\, dx$

$= \int \left(\frac{1}{2}z + \frac{1}{2}\right) . z^3 . \frac{1}{2}dz$

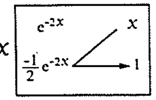
$= \int \left(\frac{1}{4}z^4 + \frac{1}{4}z^3\right) dz = \frac{1}{20}z^5 + \frac{1}{16}z^4 + c$

$= \frac{1}{20}(2x-1)^5 + \frac{1}{16}(2x-1)^4 + c$

(2) $\int x e^{-2x}\, dx$

$= \frac{-1}{2}x e^{-2x} + \int \frac{1}{2}e^{-2x}\, dx$

$= \frac{-1}{2}x e^{-2x} - \frac{1}{4}e^{-2x} + c$



[b] Let $y = \sqrt{16 + x^2}$, $z = \frac{x}{x-2}$

$\therefore \frac{dy}{dx} = \frac{2x}{2\sqrt{16 + x^2}} = \frac{x}{\sqrt{16 + x^2}}$

, $\frac{dz}{dx} = \frac{(x-2) - x}{(x-2)^2} = \frac{-2}{(x-2)^2}$

$\therefore \frac{dy}{dz} = \frac{dy}{dx} \div \frac{dz}{dx} = \frac{x}{\sqrt{16 + x^2}} \div \frac{-2}{(x-2)^2}$

$= \frac{-x(x-2)^2}{2\sqrt{16 + x^2}}$

At $x = -3$: $\frac{dy}{dz} = \frac{-(-3)(-3-2)^2}{2\sqrt{16 + (-3)^2}} = \frac{75}{10} = \frac{15}{2}$

3

[a] $x \cos y + y \cos x = 1$

(By differentiating with respect to x)

$\therefore \cos y + x(-\sin y)\frac{dy}{dx} + \frac{dy}{dx}\cos x$

$+ y(-\sin x) = 0$

$\therefore \frac{dy}{dx}(-x\sin y + \cos x) = -\cos y + y\sin x$

$\therefore \frac{dy}{dx} = \frac{y\sin x - \cos y}{\cos x - x\sin y}$

[b] $f(x) = 2x^3 + 6x^2 + 5$ $\therefore \grave{f}(x) = 6x^2 + 12x$

Putting $\grave{f}(x) = 0 : 6x^2 + 12x = 0$

$\therefore 6x(x + 2) = 0$ $\therefore x = 0 \in [-1, 1]$

or $x = -2 \notin [-1, 1]$

$f(-1) = 2(-1)^3 + 6(-1)^2 + 5 = 9$

, $f(0) = 2(0)^3 + 6(0)^2 + 5 = 5$

, $f(1) = 2(1)^3 + 6(1)^2 + 5 = 13$

The function has absoult maximum value = 13 at $x = 1$ and absoult minimum value = 5 at $x = 0$

4

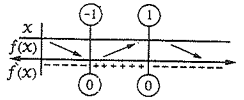
[a] (1) $f(x) = \begin{cases} 2x + x^2 & , \quad x < 0 \\ 2x - x^2 & , \quad x \geq 0 \end{cases}$

$\therefore f'(x) = \begin{cases} 2 + 2x & , \quad x < 0 \\ 2 - 2x & , \quad x > 0 \end{cases}$

$f'(0^+) = f'(0^-)$

$f'(x) = 0 : 2 + 2x = 0 \quad \therefore x = -1 < 0$

, $2 - 2x = 0 \qquad \therefore x = 1 > 0$



The function has a local minimum value at $x = -1$

, $f(-1) = 2(-1) + (-1)^2 = -1$

and has a local maximum value at $x = 1$

, $f(1) = 2(1) - (1)^2 = 1$

(2) $\int_{-1}^{3} f(x)\, dx = \int_{-1}^{0} (2x + x^2)\, dx$

$+ \int_{0}^{3} (2x - x^2)\, dx = \left[x^2 + \frac{1}{3}x^3\right]_{-1}^{0}$

$+ \left[x^2 - \frac{1}{3}x^3\right]_0^3 = \left[(0) - \left((-1)^2 + \frac{1}{3}(-1)^3\right)\right]$

$+ \left[\left((3)^2 - \frac{1}{3}(3)^3\right) - (0)\right] = \frac{-2}{3}$

[b] Volume of cube $v = x^3$

(By differentiating with respect to time)

$\therefore \frac{dv}{dt} = 3x^2 \frac{dx}{dt}$

, at $\frac{dv}{dt} = 27$ cm³/min , $x = 3$

$\therefore 27 = 3(3)^2 \cdot \frac{dx}{dt} \qquad \therefore \frac{dx}{dt} = 1$

, area of all face $A = 6x^2$

(By differentiating with respect to time)

$\therefore \frac{dA}{dt} = 12x \cdot \frac{dx}{dt}$, at $x = 3$, $\frac{dx}{dt} = 1$

$\therefore \frac{dA}{dt} = 12(3) \cdot (1) = 36$ cm²/min.

5

[a] The points of intersection $x^2 = 6x - x^2$

$2x^2 - 6x = 0$

$2x(x - 3) = 0 \qquad \therefore x = 0$ or $x = 3$

$\therefore$ The area $= \int_0^3 \left[(6x - x^2) - (x^2)\right] dx$

$= \int_0^3 (6x - 2x^2)\, dx$

$= \left[3x^2 - \frac{2}{3}x^3\right]_0^3$

$= [27 - 18] - [0] = 9$ square units.

[b] $f(x) = x^3 + ax^2 + bx$

Putting : $x = 2$, $y = 2$

$\therefore 2 = (2)^3 + a(2)^2 + b(2) \qquad \therefore 2 = 8 + 4a + 2b$

$\therefore 2a + b = -3 \qquad (1)$

, $f'(x) = 3x^2 + 2ax + b$

, $f''(x) = 6x + 2a$

Putting : $f''(x) = 0$

, $x = 2$

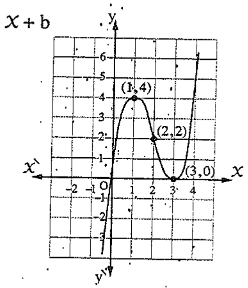
$0 = 6(2) + (2a)$

$\therefore a = -6$

Substituting in (1) :

$\therefore 2(-6) + b = -3$

$\therefore b = 9$



Model 3

1

(1) (b) (2) (b) (3) (d)

(4) (c) (5) (a) (6) (a)

2

[a] $y = x^2 \ln x$

$\therefore \frac{dy}{dx} = 2x \ln x + x^2 \times \frac{1}{x} = 2x \ln x + x$

$= x(2 \ln x + 1)$

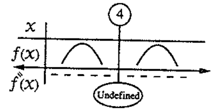
[b] $f(x) = \sqrt[3]{(x-4)^2} = (x-4)^{\frac{2}{3}}$

$\therefore f'(x) = \frac{2}{3}(x-4)^{\frac{-1}{3}}$

, $f'(4)$ undefined because it has a vertical tangent

, $f''(x) = \frac{-2}{9}(x-4)^{\frac{-4}{3}} = \frac{-2}{9\sqrt[3]{(x-4)^4}}$

, $f''(x)$ undefined at $x = 4$



The curve is convex upwards at $]-\infty, 4[$, $]4, \infty[$ and has no inflection points.

3

[a] (1) $\int x(x-5)^3\, dx$

Putting : $z = x - 5 \qquad \therefore x = z + 5$

$dx = dz$

$\therefore \int x(x-5)^3\, dx = \int (z+5)(z)^3\, dz$

$= \int (z^4 + 5z^3)\, dz$

$= \frac{1}{5}z^5 + \frac{5}{4}z^4 + c$

$= \frac{1}{5}(x-5)^5 + \frac{5}{4}(x-5)^4 + c$

Another Solution :

$\int x(x-5)^3\, dx = \int (x - 5 + 5)(x-5)^3\, dx$

$= \int \left[(x-5)^4 + 5(x-5)^3\right] dx$

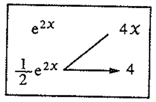
$= \frac{1}{5}(x-5)^5 + \frac{5}{4}(x-5)^4 + c$

(2) $\int 4x e^{2x}\, dx$

$= 2x e^{2x} - \int 2e^{2x}\, dx$

$= 2x e^{2x} - \frac{2e^{2x}}{2} + c$

$= 2x e^{2x} - e^{2x} + c$



[b] $f(x) = x^4 - 4x^3$

(By differentiating with respect to x)

$\therefore f'(x) = 4x^3 - 12x^2$

, $f'(x) = 0 : 4x^3 - 12x^2 = 0$

$\therefore 4x^2(x - 3) = 0$

$\therefore x = 0$ or $x = 3$

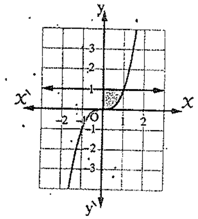
, $f(0) = 0$, $f(3) = -27$, $f(4) = 0$

f has absolute maximum value = 0

at $x = 0$, $x = 4$

4

[a]



$y_1 = 1$, $y_2 = x^3$

, $\because y_1 \geq y_2$ on $[0, 1]$

$\therefore$ The volume $= \pi \int_0^1 (y_1^2 - y_2^2)\, dx$

$= \pi \int_0^1 (1 - x^6)\, dx$

$= \pi \left[x - \frac{1}{7}x^7\right]_0^1$

$= \frac{6}{7}\pi$ cubic unit.

$\therefore \pi r^2 \times 42 = \frac{6}{7}\pi \qquad \therefore r = \frac{1}{7}$ length unit.

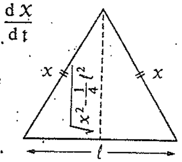
[b] Let the two equal side be x and the base ℓ

$\therefore A = \frac{1}{2}\ell \times \sqrt{x^2 - \frac{1}{4}\ell^2}$

(By differentiating with respect to time)

$\frac{dA}{dt} = \frac{1}{2}\ell \times \frac{2x}{2\sqrt{x^2 - \frac{1}{4}\ell^2}} \cdot \frac{dx}{dt}$

$= \frac{\ell x}{2\sqrt{x^2 - \frac{1}{4}\ell^2}} \cdot \frac{dx}{dt}$



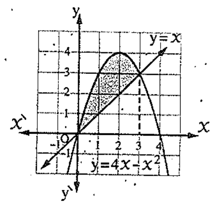
putting $\frac{dx}{dt} = -3$ cm./min. , $x = \ell$

$\therefore \frac{dA}{dt} = \frac{\ell^2}{2\sqrt{\ell^2 - \frac{1}{4}\ell^2}}(-3) = -\sqrt{3}\,\ell$

$\therefore$ The rate of decrease $= \sqrt{3}\,\ell$ cm²/min.

5

[a]



$\because x - y = 0$, $y = x$

The points of intersection : $x = 4x - x^2$

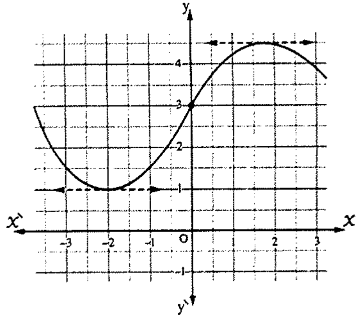
$\therefore 3x - x^2 = 0 \quad \therefore x(3 - x) = 0 \quad \therefore x = 0$ or $x = 3$

Area a $= \int_0^3 \left[(4x - x^2) - (x)\right] dx$

$= \int_0^3 (3x - x^2)\, dx = \left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^3$

$= \left[\frac{3}{2}(3)^2 - \frac{1}{3}(3)^3\right] - [0] = \frac{9}{2}$ square unit.

[b]



Model 4

1

(1) (b)	(2) (b)	(3) (c)
(4) (d)	(5) (c)	(6) (c)

2

[a] (1) $\int (3x^2 - 4e^{2x})\, dx = x^3 - \frac{4e^{2x}}{2} + c$

$= x^3 - 2e^{2x} + c$

(2) $\int \frac{x - 1}{\sqrt{x + 3}}\, dx$

Let $z = x + 3 \qquad \therefore x = z - 3$

$, dx = dz \quad , x - 1 = z - 4$

$\therefore$ The integration $= \int \frac{z - 4}{\sqrt{z}}\, dz$

$= \int \left(z^{\frac{1}{2}} - 4z^{\frac{-1}{2}}\right) dz = \frac{2}{3}z^{\frac{3}{2}} - 8z^{\frac{1}{2}} + c$

$= \frac{2}{3}z^{\frac{1}{2}}[z - 12] + c$

$= \frac{2}{3}\sqrt{x + 3}\,[x + 3 - 12] + c$

$= \frac{2}{3}\sqrt{x + 3}\,[x - 9] + c$

[b] $\sin y + \cos 2x = 0$

(By differentiating with respect to x)

$\cos y \frac{dy}{dx} - 2\sin 2x = 0$

(By differentiating with respect to x another time)

$\therefore \cos y \frac{d^2y}{dx^2} + \frac{dy}{dx} \times -\sin y$

$\times \frac{dy}{dx} - 4\cos 2x = 0$ (Dividing by cos y)

$\therefore \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 \tan y = 4\cos 2x \sec y$

3

[a] $\int_1^4 [f(x) + 2g(x) - 4]\, dx$

$= \int_1^4 f(x)\, dx + 2\int_1^4 g(x)\, dx - 4\int_1^4 dx$

$= 7 + 2 \times -3 - 4[x]_1^4 = 7 + (-6) - 4[4 - 1] = -11$

[b] $f(x) = ax^3 + bx^2 + cx + d$

$\therefore f'(x) = 3ax^2 + 2bx + c$

$, f''(x) = 6ax + 2b$

$\because$ The curve has inflection point at (1 , 2)

$\therefore f''(1) = 0 \qquad \therefore 6a + 2b = 0$

$\therefore b = -3a$ (1)

$\because$ The curve has a local maximum value at (2 , 4)

$\therefore f'(2) = 0 \qquad \therefore 12a + 4b + c = 0$

From (1) : $\qquad \therefore 12a - 12a + c = 0$

$\therefore c = 0$ (2)

$\because$ The curve passes through (1 , 2)

$\therefore f(1) = 2 \qquad \therefore a + b + c + d = 2$

From (1) and (2) : $\qquad \therefore a - 3a + d = 2$

$\therefore -2a + d = 2$ (3)

$\because$ The curve passes through (2 , 4)

$\therefore f(2) = 4 \qquad \therefore 8a + 4b + 2c + d = 4$

From (1) , (2) : $\qquad \therefore 8a - 12a + 0 + d = 4$

$\therefore -4a + d = 4$ (4)

By solving the two equations (3) and (4) :

$\therefore a = -1 \quad , \quad d = 0 \quad \therefore b = 3$

$\therefore$ The equation of the curve is :

$f(x) = -x^3 + 3x^2$

4

[a] $\sqrt{x} + \sqrt{y} = 1 \qquad \therefore \sqrt{y} = 1 - \sqrt{x}$

$\therefore y = 1 - 2\sqrt{x} + x$

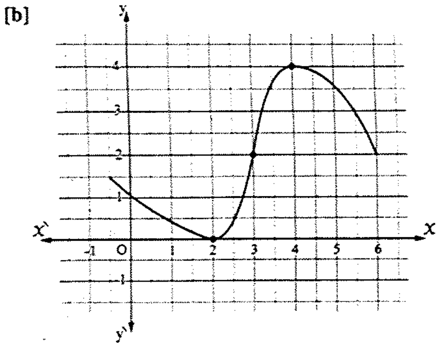
, at $x = 0 \qquad \therefore y = 1$

, at $y = 0 \qquad \therefore x = 1$

$\therefore$ Area $= \int_0^1 (1 - 2x^{\frac{1}{2}} + x)\, dx$

$= \left[x - \frac{4}{3}x^{\frac{3}{2}} + \frac{1}{2}x^2\right]_0^1 = \frac{1}{6}$ square unit.

[b]



5

[a] The point of intersection

$y_1 = \frac{4}{x} \quad , y_2 = 5 - x$

$\frac{4}{x} = 5 - x \qquad \therefore 4 = 5x - x^2$

$\therefore x^2 - 5x + 4 = 0 \qquad \therefore x = 1$ or $x = 4$

$\therefore v = \pi \int_1^4 (y_2^2 - y_1^2)\, dx$

$= \pi \int_1^4 \left((5 - x)^2 - (4x^{-1})^2\right) dx$

$= \pi \int_1^4 \left((25 - 10x + x^2) - (16x^{-2})\right) dx$

$= \pi \left[25x - 5x^2 + \frac{1}{3}x^3 + 16x^{-1}\right]_1^4$

$= \pi \left[(100 - 80 + \frac{1}{3}(64) + 4)\right.$

$\left. - (25 - 5 + \frac{1}{3} + 16)\right] = 9\pi$ cubic unit

[b] $A = \pi(r_2^2 - r_1^2)$

(By differentiating with respect to time)

$\therefore \frac{dA}{dt} = \pi\left(2r_2 \frac{dr_2}{dt} - 2r_1 \frac{dr_1}{dt}\right)$

$, r_1 = 6$ cm. $, \frac{dr_1}{dt} = 0.3$ cm./sec.

$, r_2 = 10$ cm. $, \frac{dr_2}{dt} = -0.2$ cm./sec.

$\therefore \frac{dA}{dt} = \pi\left(2(10)(-0.2) - 2(6)(0.3)\right) = -7.6\pi$

$\therefore$ The rate of decrease of area (A) included between the two circles $= 7.6\pi$ cm^2/sec.

Model 5

1

(1) $]-\infty , 0[\cup]2 , \infty[$	(2) $\{0 , 2\}$
(3) $]1 , \infty[$	(4) 0
(5) 2	(6) 4

2

[a] (1) $\int \csc^2\left(\frac{x + 5}{2}\right) dx = -2\int \frac{-1}{2}\csc^2\left(\frac{x + 5}{2}\right) dx$

$= -2\cot\left(\frac{x + 5}{2}\right) + c$

(2) $\int \frac{5x}{3x^2 - 1}\, dx = \frac{5}{6}\int \frac{6x}{3x^2 - 1}\, dx$

$= \frac{5}{6}\ln|3x^2 - 1| + c$

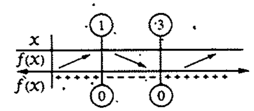
[b] (1) $f(x) = x^3 - 6x^2 + 9x - 1$

$\therefore f'(x) = 3x^2 - 12x + 9$

Putting : $f'(x) = 0 : \therefore 3x^2 - 12x + 9 = 0$

$\therefore 3(x^2 - 4x + 3) = 0$

$\therefore 3(x - 3)(x - 1) = 0 \qquad \therefore x = 3 , x = 1$



f is increasing on $]-\infty , 1[,]3 , \infty[$

and decreasing on $]1 , 3[$

(2) $f(0) = -1 \quad , \quad f(1) = 3 \quad , \quad f(2) = 1$

$\therefore f$ has absolute maximum value $= 3$ at $x = 1$

3

[a] $y = 4 + \cot x - \sec^2 x$

$\therefore \frac{dy}{dx} = -\csc^2 x - 2\sec x \cdot \sec x \tan x$

$= -\csc^2 x - 2\sec^2 x \tan x$

$\therefore \left(\frac{dy}{dx}\right)_{x = \frac{\pi}{4}} = -6$

$\therefore$ The slope of tangent $= -6$

$\therefore$ The slope of normal $= \frac{1}{6}$

$\because$ at $x = \frac{\pi}{4} \qquad \therefore y = 3 \qquad \therefore$ The point $\left(\frac{\pi}{4} , 3\right)$

$\therefore$ The equation of normal is : $(y - 3) = \frac{1}{6}\left(x - \frac{\pi}{4}\right)$

$\therefore 4x - 24y - \pi + 72 = 0$

[b] $\frac{dv}{dt} = 2t + 3$ (By integration)

$\therefore \int dv = \int (2t+3)\,dt \qquad \therefore v = t^2 + 3t + c$

at $t = 0$, $v = 0$ (Tank is empty)

$\therefore c = 0 \qquad \therefore v = t^2 + 3t$

, at $v = 10 : 10 = t^2 + 3t$

$\therefore t^2 + 3t - 10 = 0 \qquad \therefore (t+5)(t-2) = 0$

$\therefore t = -5$ (refused) or $t = 2$

$\therefore$ The tank is filled after 2 minutes.

4

[a] $\lim\limits_{x \to \infty} \left(\frac{2x-1}{2x+1}\right)^{2x} = \lim\limits_{x \to \infty} \left(1 - \frac{2}{2x+1}\right)^{2x}$

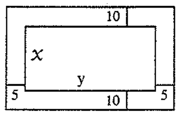
Let $y = \frac{-2}{2x+1}$, thus $x = \frac{-1}{y} - \frac{1}{2}$

$\because x \longrightarrow \infty \qquad \therefore y \longrightarrow 0$

$\therefore \lim\limits_{y \to 0} (1+y)^{\frac{-2}{y} - 1} = \lim\limits_{y \to 0} \left((1+y)^{\frac{1}{y}}\right)^{-2} \times (1+y)^{-1}$

$\therefore e^{-2} \times 1^{-1} = \frac{1}{e^2}$

[b] Area of sticker $= 800 = x\,y$



$\therefore y = \frac{800}{x}$

Area of rectangle

$= (x + 20)(y + 10)$

$\therefore A = xy + 10x + 20y + 200$

$= 800 + 10x + 20 \times \frac{800}{x} + 200$

$\therefore \acute{A} = 10 - 16000\,x^{-2}$

, $\acute{\acute{A}} = 32000\,x^{-3}$

Putting : $\acute{A} = 0 \quad \therefore x = 40 \quad , \because \acute{\acute{A}}_{x=40} > 0$

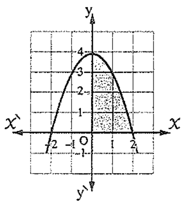
$\therefore x = 40$ makes the area as small as possible.

$\therefore$ Dimensions of rectangle $= 40 + 20 = 60$ cm.

, $\frac{800}{40} + 10 = 30$ cm..

5

[a]



$y = 4 - x^2 \qquad \because$ Rotation about the x-axis

$\therefore$ The volume $= \pi \int_0^2 y^2\,dx$

$= \pi \int_0^2 (4 - x^2)^2\,dx$

$= \pi \int_0^2 (16 - 8x^2 + x^4)\,dx$

$= \pi \left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right]_0^2$

$= \frac{256}{15}\pi$ cubic unit.

[b] $f(x) = x^3 + a x^2 + b x + 4$

$\therefore \acute{f}(x) = 3x^2 + 2ax + b$

, $\acute{\acute{f}}(x) = 6x + 2a \qquad , \because \acute{f}(x) = 0$ at $x = 2$

$\therefore 0 = 3(2)^2 + 2a(2) + b$

$\therefore 4a + b = -12$ (1)

, $\acute{\acute{f}}(x) = 0$ at $x = 1$

$\therefore 0 = 6(1) + 2a \qquad \therefore a = -3$ (2)

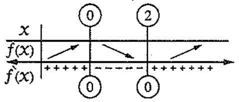
From (1) , (2) : $\qquad \therefore b = 0$

$\therefore f(x) = x^3 - 3x^2 + 4$

, putting $\acute{f}(x) = 0$

$\therefore 3x^2 - 6x = 0$

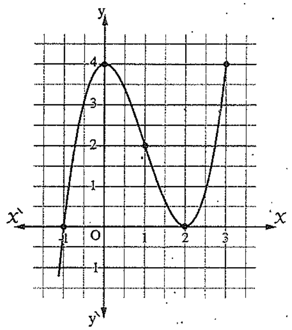
$\therefore x = 0$ or $x = 2$



* we find some assistance points :

$f(-1) = 0$, $f(3) = 4$

x	-1	0	1	2	3
$f(x)$	0	4	2	0	4



Model 6

1

(1) (b)	(2) (a)	(3) (a)
(4) (b)	(5) (a)	(6) (b)

2

[a] (1) $\int \frac{7x^3}{2-5x^4}\,dx = \frac{-7}{20}\int \frac{-20x^3}{2-5x^4}\,dx$

$= \frac{-7}{20}\ln|2 - 5x^4| + c$

(2) $\int \left(3e^{-5x} + \frac{\pi}{x}\right) dx = \frac{-3}{5}e^{-5x} + \pi \ln|x| + c$

[b] $y = a\,e^{x^2+1} \qquad \therefore \frac{dy}{dx} = 2a\,x\,e^{x^2+1}$

, $\frac{d^2y}{dx^2} = 2a\,x \,.\, 2x\,e^{x^2+1} + 2a\,e^{x^2+1}$

$= 4a\,x^2 e^{x^2+1} + 2a\,e^{x^2+1}$

$= 2a\,e^{x^2+1}[2x^2 + 1]$

, $\frac{d^3y}{dx^3} = 2a\,e^{x^2+1}[4x] + 4a\,x\,e^{x^2+1}[2x^2+1]$

$= 2a\,e^{x^2+1}\left[4x + 2x[2x^2+1]\right]$

$= 2a\,e^{x^2+1}[4x + 4x^3 + 2x]$

$= 2y[4x^3 + 6x] = 4y\,x[2x^2 + 3]$

3

[a] $\int \cot x \csc^3 x\,dx = \int -\csc^2 x \,.\, (-\csc x \cot x)\,dx$

$= \frac{-1}{3}\csc^3 x + c$

[b] $s = \sqrt{(x-1)^2 + (y-0)^2}$

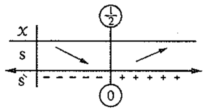
, $\because y = \sqrt{x} \qquad \therefore y^2 = x$

$\therefore s = \sqrt{x^2 - 2x + 1 + y^2} = \sqrt{x^2 - 2x + 1 + x}$

$= \sqrt{x^2 - x + 1}$

$\therefore \frac{ds}{dx} = \frac{2x - 1}{2\sqrt{x^2 - x + 1}}$

Putting : $\frac{ds}{dx} = 0 \qquad \therefore 2x - 1 = 0 \qquad \therefore x = \frac{1}{2}$



$\therefore$ at $x = \frac{1}{2}$

$\therefore$ s has a minimum value.

$\therefore y = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} \qquad \therefore$ The point is : $\left(\frac{1}{2} , \frac{1}{\sqrt{2}}\right)$

4

[a] $f(x) = \begin{cases} -x^2 + 4x & , \quad -1 \le x \le 0 \\ x^2 - 4x & , \quad 0 < x \le 3 \end{cases}$

, $\because \acute{f}(0^+) \ne \acute{f}(0^-)$

$\because \acute{f}(0)$ is not exist.

$\therefore \acute{f}(x) = \begin{cases} -2x + 4 & , \quad -1 \le x < 0 \\ \text{is not exist} & , \quad x = 0 \\ 2x - 4 & , \quad 0 < x \le 3 \end{cases}$

, $\acute{f}(x) = 0$ in $-1 \le x < 0$

$\therefore -2x + 4 = 0 \quad , x = 2 \notin$ interval

, $\acute{f}(x) = 0$ in $0 < x \le 3$

$\therefore 2x - 4 = 0 \qquad \therefore x = 2 \in$ interval

$\therefore f(-1) = -5$, $f(2) = -4$, $f(3) = -3$

, $f(0) = 0$

$\therefore f$ has absolute maximum value = zero at $x = 0$ and absolute minimum value $= -5$ at $x = -1$

[b] $\because \frac{dy}{dx} = 6x^2 + b\,x \qquad \therefore y = 2x^3 + \frac{b}{2}x^2 + c$

Putting : $x = 0$, $y = 5 \quad \therefore c = 5$

$\therefore y = 2x^3 + \frac{b}{2}x^2 + 5$

Putting : $x = 2$, $y = -3$

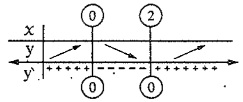
$\therefore -3 = 2(2)^3 + \frac{b}{2}(2)^2 + 5$

$\therefore -3 = 21 + 2b \qquad \therefore b = -12$

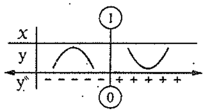
$\therefore f(x) = 2x^3 - 6x^2 + 5$

$\therefore \acute{f}(x) = 6x^2 - 12x \qquad \therefore \acute{\acute{f}}(x) = 12x - 12$

put $\acute{f}(x) = 0 \qquad \therefore x = 0$ or 2

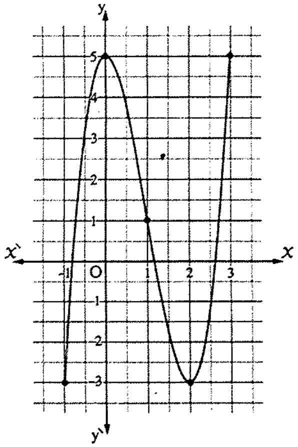


* Put $\acute{\acute{f}}(x) = 0 \qquad \therefore x = 1$



* Determine other points : $f(-1) = -3$, $f(3) = 5$

x	-1	0	1	2	3
$f(x)$	-3	5	1	-3	5



5

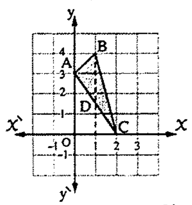
[a] let $y = \ln(9 + x^3)$ $\qquad \therefore \frac{dy}{dx} = 3x^2 \times \frac{1}{9+x^3}$

let $z = x^2 + 3$ $\qquad \therefore \frac{dz}{dx} = 2x$

$\therefore \frac{dy}{dx} = \frac{dy}{dx} \div \frac{dz}{dx} = \frac{3x^2}{9+x^3} \times \frac{1}{2x} = \frac{3}{2} \times \frac{x}{9+x^3}$

$\therefore \left(\frac{dy}{dx}\right)_{x=1} = \frac{3}{2}\left(\frac{1}{10}\right) = \frac{3}{20}$

[b] The equation of $\overleftrightarrow{AB}$: $\frac{y-3}{x-0} = \frac{4-3}{1-0} = \frac{1}{1}$



$\therefore y - 3 = x$ $\qquad \therefore y = x + 3$

The equation of $\overleftrightarrow{BC}$: $\frac{y-0}{x-2} = \frac{4-0}{1-2} = -4$

$\therefore y = -4x + 8$

The equation of $\overleftrightarrow{AC}$: $\frac{y-3}{x-0} = \frac{0-3}{2-0} = \frac{-3}{2}$

$\therefore y = \frac{-3}{2}x + 3$

$\therefore$ Area of ΔABC = area of ΔABD + area of ΔDBC

$= {}_0\int^1 \left[(x+3) - \left(\frac{-3}{2}x + 3\right)\right] dx + {}_1\int^2 \left[(-4x+8) - \left(\frac{-3}{2}x+3\right)\right] dx$

$= {}_0\int^1 \frac{5}{2} x\, dx + {}_1\int^2 \left(\frac{-5}{2}x + 5\right) dx = \left[\frac{5}{4}x^2\right]_0^1$

$+ \left[\frac{-5}{4}x^2 + 5x\right]_1^2 = \frac{5}{4} + \frac{5}{4} = \frac{5}{2}$ square unit.

If $y = \frac{-3}{2}x + 3$ $\qquad \therefore x = \frac{-2}{3}y + 2$

$\therefore$ The volume from rotation ΔAOC about y-axis

$= \pi\, {}_0\int^3 x^2\, dy = \pi\, {}_0\int^3 \left(\frac{-2}{3}y + 2\right)^2 dy$

$= \pi \left[\frac{-1}{2}\left(\frac{-2}{3}y + 2\right)^3\right]_0^3 = 4\pi$ cubic units.

Model 7

1

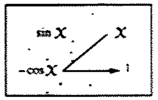
(1) (b)	(2) (a)	(3) (a)
(4) (b)	(5) (a)	(6) (b)

2

[a] (1) $\int x \sin x\, dx$

$= -x\cos x - \int -\cos x\, dx$

$= -x\cos x + \sin x + c$



(2) $\because f(x) = \sqrt{x^4 + x^2}$ even function

$\therefore {}_{-1}\int^1 f(x)\, dx = 2\, {}_0\int^1 \sqrt{x^4 + x^2}\, dx$

$= 2\, {}_0\int^1 |x| \sqrt{x^2+1}\, dx$

$= \left[\frac{2}{3}(x^2+1)^{\frac{3}{2}}\right]_0^1$

$= \frac{2}{3}\left[2\sqrt{2} - 1\right] = \frac{4\sqrt{2}-2}{3}$

[b] $y = \ln\left(2 - \sqrt{2}\cos x\right)$, at $x = \frac{\pi}{4}$, then $y = 0$

, $\frac{dy}{dx} = \frac{\sqrt{2}\sin x}{2 - \sqrt{2}\cos x}$

$\therefore$ Slope of tangent $= \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = 1$

$\therefore$ Equation of tangent is : $\frac{y-0}{x-\frac{\pi}{4}} = 1$

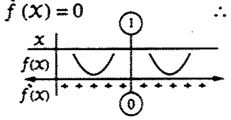
$\therefore y = x - \frac{\pi}{4}$

3

[a] $f(x) = (x-1)^4 + 3$ $\qquad \therefore f'(x) = 4(x-1)^3$

, $f''(x) = 12(x-1)^2$

Putting : $f''(x) = 0$ $\qquad \therefore x = 1$



The curve is convex downwards at $]-\infty, 1[$, $]1, \infty[$ and has no inflection points.

[b] Let the dimensions of cuboid are : (x, x, z)

$\because \frac{dx}{dt} = 0.4$ cm./sec. $\quad , \frac{dz}{dt} = -0.5$ cm./sec.

$\therefore$ volume $(v) = x^2 z$

$\therefore \frac{dv}{dt} = x^2 \frac{dz}{dt} + 2x\frac{dx}{dt} \cdot z$

$= (6)^2 \times -0.5 + 2(6)(0.4)(5) = 6$ cm.^{3}/sec.

4

[a] Put $z = x + 1$ $\qquad \therefore x = z - 1$

$\therefore dx = dz$ at $x = 0$ $\qquad \therefore z = 1$

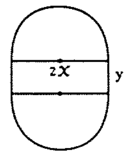
, at $x = 3$ $\qquad \therefore z = 4$

${}_1\int^4 (z-1)\, z^{\frac{1}{2}}\, dz = {}_1\int^4 \left(z^{\frac{3}{2}} - z^{\frac{1}{2}}\right) dz$

$= \left[\frac{2}{5}z^{\frac{5}{2}} - \frac{2}{3}z^{\frac{3}{2}}\right]_1^4$

$= \left[\frac{2}{5}(4)^{\frac{5}{2}} - \frac{2}{3}(4)^{\frac{3}{2}}\right]$

$- \left[\frac{2}{5} - \frac{2}{3}\right] = \frac{116}{15}$



[b] The perimeter = 400

$\therefore 2y + 2\pi x = 400$

$\therefore y = 200 - \pi x$

, let the area of playground (A) :

$\therefore A = 2xy + \pi x^2 = 2x(200 - \pi x) + \pi x^2$

$= 400x - 2\pi x^2 + \pi x^2 = 400x - \pi x^2$

$\therefore A' = 400 - 2\pi x$, putting : $A' = 0$

$\therefore x = \frac{200}{\pi}$, $A'' = -2\pi < 0$ maximum.

In the case of area is great as possible.

$\therefore x = \frac{200}{\pi}$ $\qquad \therefore y = 200 - \pi \times \frac{200}{\pi} = 0$

$\therefore$ The figure becomes a circle with radius length $\frac{200}{\pi}$

5

[a] $f(x) = x^3 - 3x + 3$

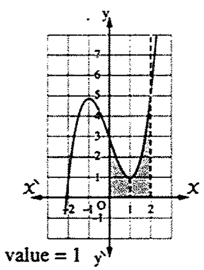
$\therefore f'(x) = 3x^2 - 3$

, Putting : $f'(x) = 0$

$\therefore x = 1, -1$

, $f(0) = 3$, $f(1) = 1$

$f(2) = 5$



$\therefore$ The absolute minimum value = 1

, the absolute maximum value = 5

The area $= {}_0\int^2 (x^3 - 3x + 3)\, dx$

$= \left[\frac{1}{4}x^4 - \frac{3}{2}x^2 + 3x\right]_0^2$

= 4 square units.

[b] The volume $= \pi\, {}_1\int^2 (2x^{-1})^2\, dx = \pi\, {}_1\int^2 4x^{-2}\, dx$

$= \pi\left[-4x^{-1}\right]_1^2$

$= \pi\left[(-4(2)^{-1}) - (-4)\right]$

$= 2\pi$ cubic units.

Model 8

1

(1) $\frac{-3}{2}$	(2) $7\sec x \tan x\, e^{\sec x}$
(3) $(0, -1)$	(4) ${}_2\int^4 f(x)\, dx$
(5) $\frac{32}{3}$	(6) $\frac{1}{2}$

2

[a] (1) $\int \frac{(x+3)^3 - 27}{x}\, dx$

$= \int \frac{x^3 + 9x^2 + 27x + 27 - 27}{x}\, dx$

$= \int (x^2 + 9x + 27)\, dx$

$= \frac{1}{3}x^3 + \frac{9}{2}x^2 + 27x + c$

(2) $\int x^2 e^{-x}\, dx$

$= -x^2 e^{-x} - \int -2x e^{-x}\, dx$

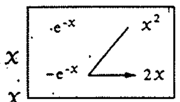
$= -x^2 e^{-x} + \int 2x e^{-x}\, dx$

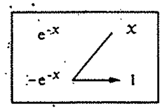
$= -x^2 e^{-x} + 2$

$\left[-x e^{-x} - \int -e^{-x}\, dx\right]$

$= -x^2 e^{-x} - 2x e^{-x}$

$-2e^{-x} + c = -e^{-x}\left[x^2 + 2x + 2\right] + c$





[b] $f\left(\frac{\pi}{4}\right) = 2\tan^3\left(\frac{\pi}{4}\right) = 2$ $\qquad \therefore$ The point $\left(\frac{\pi}{4}, 2\right)$

, $f'(x) = 6\tan^2 x \cdot \sec^2 x$

$\therefore f'\left(\frac{\pi}{4}\right) = 6\tan^2\frac{\pi}{4} \cdot \sec^2\frac{\pi}{4} = 12$

$\therefore$ Equation of tangent is : $y - 2 = 12\left(x - \frac{\pi}{4}\right)$

$\therefore y = 12x - 3\pi + 2$

3

[a] $\int_0^5 |x-2|\, dx$

$f(x) = |x-2| = \begin{cases} x-2 & , \quad x \geq 2 \\ -x+2 & , \quad x < 2 \end{cases}$

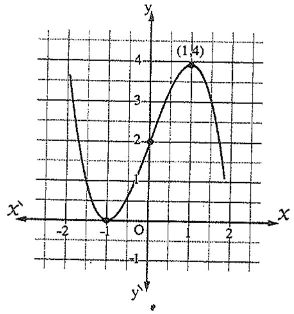
$\therefore \int_0^5 f(x)\, dx = \int_0^2 (-x+2)\, dx + \int_2^5 (x-2)\, dx$

$= \left[\frac{-1}{2}x^2 + 2x\right]_0^2 + \left[\frac{1}{2}x^2 - 2x\right]_2^5$

$= \left[(-2+4)-0\right] + \left[\left(\frac{1}{2}(25) - 2(5)\right)\right]$

$- \left[\frac{1}{2}(4) - 2(2)\right] = \frac{13}{2}$

[b]



4

[a] $f(x) = 3\sqrt{4-x^2}$

$\therefore \acute{f}(x) = \left(\frac{3}{2}(4-x^2)^{\frac{-1}{2}}\right)(-2x) = \frac{-3x}{\sqrt{4-x^2}}$

$, \acute{f}(x) = 0$ at $x = 0 \in [0, 2]$

$, \acute{f}(x)$ is undefined at $x = 2 \in [0, 2]$

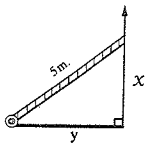
$, x = -2 \notin [0, 2]$ $\quad , f(0) = 6 \quad , f(2) = 0$

$\therefore f$ has absolute maximum value $= 6$ at $x = 0$

, has absolute miniumum value $= 0$ at $x = 2$

[b] $\frac{dx}{dt} = 1$ m./min.

, y the length of projection of the rod on the floor.



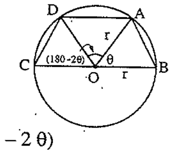
$\therefore x^2 + y^2 = 25$

$\therefore 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$\therefore \frac{dy}{dt} = \frac{-x}{y}\frac{dx}{dt}$ $\quad$, at $x = 3$ m. $\quad \therefore y = 4$

$\therefore \frac{dy}{dt} = \frac{-3}{4} \times 1 = \frac{-3}{4}$ m./min.

5



[a] Area of trapezoid

$= A(\Delta OBA) + A(\Delta OAD)$

$+ A(\Delta ODC)$

$= \frac{1}{2}r^2 \sin\theta + \frac{1}{2}r^2 \sin(180° - 2\theta)$

$+ \frac{1}{2}r^2 \sin\theta = r^2 \sin\theta + \frac{1}{2}r^2 \sin 2\theta$

$\therefore \grave{A} = r^2 \cos\theta + r^2 \cos 2\theta$

$= r^2 \cos\theta + (2\cos^2\theta - 1) r^2$

, putting : $\grave{A} = 0$

$\therefore r^2 (2\cos^2\theta + \cos\theta - 1) = 0$

$\therefore r^2 (2\cos\theta - 1)(\cos\theta + 1)$

$\therefore \cos\theta = \frac{1}{2}$

or $\cos\theta = -1$ (refused)

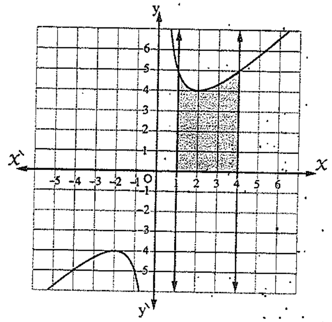
$\therefore \theta = 60°$

$, \ddot{A}_{\theta = 60} > 0$ maximum

$\therefore$ The base angle ($\angle ABC$)

$= \frac{180° - 60°}{2} = 60°$

[b]



$xy = 4 + x^2$ $\qquad \therefore y = \frac{4}{x} + x$

(1) Area $= \int_1^4 \left(\frac{4}{x} + x\right) dx$

$= \left[4\ln|x| + \frac{1}{2}x^2\right]_1^4$

$= 4\ln 4 + \frac{15}{2}$ square unit.

(2) Volume $= \pi \int_1^4 \left(x + \frac{4}{x}\right)^2 dx$

$= \pi \int_1^4 \left(\frac{16}{x^2} + 8 + x^2\right) dx$

$= \pi \left[-16x^{-1} + 8x + \frac{1}{3}x^3\right]_1^4$

$= 57\pi$ cubic unit.

Model 9

1

(1) (a)	(2) (d)	(3) (a)
(4) (b)	(5) (c)	(6) (c)

2

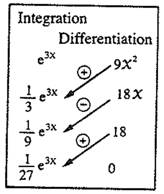
[a] (1) $\int \frac{3x}{x^2-1}\, dx = \frac{3}{2}\int \frac{2x}{x^2-1}\, dx$

$= \frac{3}{2}\ln|x^2 - 1| + c$

(2) $\int 9x^2 e^{3x}\, dx$

$= 3x^2 e^{3x} - 2x e^{3x}$

$+ \frac{2}{3}e^{3x} + c$



[b] $y^3 = x^2$ $\qquad \therefore y = x^{\frac{2}{3}}$

$\therefore \frac{dy}{dx} = \frac{2}{3}x^{\frac{-1}{3}} = \frac{2}{3\sqrt[3]{x}}$

$\therefore$ Slope of tangent at $(x = 8) = \left(\frac{dy}{dx}\right)_{x=8} = \frac{1}{3}$

$\therefore \tan\theta = \frac{1}{3}$ $\qquad \therefore \theta \simeq 18° 26$

3

[a] $\because \sin x = xy$

(By differentiating with respect to x)

$\therefore \cos x = x\acute{y} + y$ $\qquad$ (1)

$\therefore \acute{y} = \frac{\cos x - y}{x}$

(By differentiating (1) with respect to x another time)

$\therefore -\sin x = x\ddot{y} + \acute{y} + \acute{y}$

$\therefore -\sin x = x\ddot{y} + 2\acute{y}$

$\therefore -xy = x\ddot{y} + 2 \times \left(\frac{\cos x - y}{x}\right)$

$\therefore -x^2 y = x^2\ddot{y} + 2\cos x - 2y$

$\therefore x^2(y + \ddot{y}) + 2\cos x = 2y$

[b] $y = 2x^3 + 3x^2 + 4x + 5$

$\therefore \frac{dy}{dx} = 6x^2 + 6x + 4$

$\therefore$ Slope of tangent at $(-1, 2) = 4$

Putting : $\frac{dy}{dx} = 4$

$\therefore 6x^2 + 6x + 4 = 4$ $\qquad \therefore 6x(x+1) = 0$

$\therefore x = 0$ thus $y = 5$ or $x = -1$ thus $y = 2$

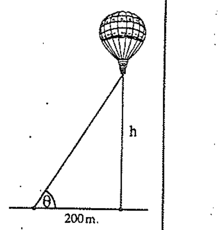
$\therefore$ The other point which its tangent has the same slope is $(0, 5)$

eqution of other tangent : $\frac{y-5}{x-0} = 4$

$\therefore 4x - y + 5 = 0$

4

[a] $\tan\theta = \frac{h}{200}$



$\therefore h = 200\tan\theta$

$\therefore \frac{dh}{dt} = 200\sec^2\theta\, \frac{d\theta}{dt}$

at $h = 200$ $\quad , \theta = \frac{\pi}{4}$

$\therefore 28 = 200\sec^2\frac{\pi}{4} \times \frac{d\theta}{dt}$

$\therefore 28 = 200 \times 2 \times \frac{d\theta}{dt}$

$\therefore \frac{d\theta}{dt} = \frac{28}{400} = 0.07$ rad./min.

[b] $\frac{dy}{dx} = 3x^2 - 3$

$\therefore y = \int (3x^2 - 3)\, dx = x^3 - 3x + c$

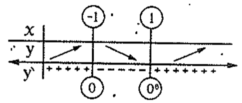
at $x = -2$ $\qquad \therefore y = -1$

$\therefore -1 = (-2)^3 - 3(-2) + c$

$\therefore c = 1$ $\qquad \therefore y = x^3 - 3x + 1$

Putting : $\frac{dy}{dx} = 0$ $\qquad \therefore 3(x^2 - 1) = 0$

$\therefore x = \pm 1$

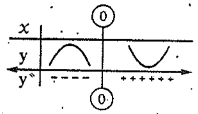


$\therefore f(-1) = 3$ local maximum value

$, f(1) = -1$ local minimum value

$, \frac{d^2y}{dx^2} = 6x$ $\qquad$, put $\frac{d^2y}{dx^2} = 0$

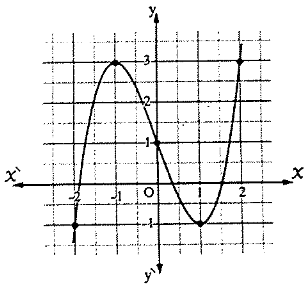
$\therefore x = 0$



$\therefore (0, 1)$ inflection point , finding helping point

$f(2) = 3$

x	-2	-1	0	1	2
y	-1	3	1	-1	3



5

(1) The equation of $\overleftrightarrow{AB}$: $\frac{y-2}{x-0} = \frac{4-2}{6-0} = \frac{1}{3}$

$\therefore y = \frac{1}{3}x + 2$

(2) The curve f cuts $\overleftrightarrow{AB}$ when $\frac{9}{x} = \frac{1}{3}x + 2$

$\therefore 27 = x^2 + 6x$

$\therefore x^2 + 6x - 27 = 0$

$\therefore (x+9)(x-3) = 0$ $\qquad \therefore x = -9$ (refused)

or $x = 3$ $\qquad \therefore y = 3$ $\qquad \therefore C = (3, 3)$

(3) Slope of tangent of curve at any point :

$\frac{dy}{dx} = \frac{-9}{x^2}$

$\therefore$ Slope of tangent at $C = -1$

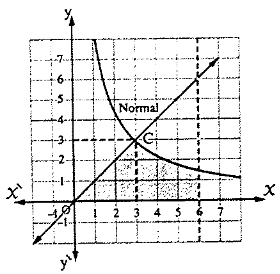
$\therefore$ Slope of normal $= 1$

$\therefore$ Equation of normal : $\frac{y-3}{x-3} = 1$

$\therefore x - y = 0$

$\therefore$ It passes through origin point.

(4)



The volume $= \pi \int_0^3 x^2\,dx + \pi \int_3^6 \left(\frac{9}{x}\right)^2 dx$

$= \pi \left[\frac{1}{3}x^3\right]_0^3 + \left[-81\,x^{-1}\right]_3^6$

$= 9\pi + \frac{27}{2}\pi = \frac{45}{2}\pi$ cubic unit.

Model 10

1

(1) e $\qquad$ (2) $6\csc^2 x\,(5 - 2\cot x)^2$

(3) 3 $\qquad$ (4) 44

(5) 0 $\qquad$ (6) $\frac{16}{15}$

2

[a] (1) $\int \tan(3x+1)\,dx = \frac{-1}{3}\int \frac{-3\sin(3x+1)}{\cos(3x+1)}\,dx$

$= \frac{-1}{3}\ln|\cos(3x+1)| + c$

$= \frac{1}{3}\ln|\sec(3x+1)| + c$

(2) $\int (1-x^2)(3x-x^3)^5\,dx$

$= \frac{1}{3}\int (3-3x^2)(3x-x^3)^5\,dx$

$= \frac{1}{3} \times \frac{(3x-x^3)^6}{6} + c = \frac{1}{18}(3x-x^3)^6 + c$

[b] (1) $\because x = 2n^3 + 3$ $\qquad \therefore \frac{dx}{dn} = 6n^2$

$, y = n^4$ $\qquad \therefore \frac{dy}{dn} = 4n^3$

at $n = 1$: $x = 2(1)^3 + 3 = 5$

$, y = (1)^4 = 1$ $\qquad \therefore$ The point $(5, 1)$

$, \frac{dy}{dx} = \frac{dy}{dn} \div \frac{dx}{dn} = \frac{4n^3}{6n^2} = \frac{2n}{3}$

$\therefore$ At $n = 1$: $\frac{dy}{dx} = \frac{2(1)}{3} = \frac{2}{3}$

$\therefore$ Equation of tangent : $y - 1 = \frac{2}{3}(x-5)$

$\therefore 2x - 3y - 7 = 0$

(2) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dn}\left(\frac{dy}{dx}\right) \times \frac{dn}{dx}$

$= \frac{d}{dn}\left(\frac{2n}{3}\right) \times \frac{1}{6n^2} = \frac{2}{3} \times \frac{1}{6n^2} = \frac{1}{9n^2}$

$\therefore$ at $n = 1$ $\qquad \therefore \frac{d^2y}{dx^2} = \frac{1}{9}$

3

[a] $f(x) = |x^3 - 1| = \begin{cases} -x^3 + 1 & , \quad x < 1 \\ x^3 - 1 & , \quad x \geq 1 \end{cases}$

$f'(1^-) = \lim_{h \to 0^-} \frac{-(1+h)^3 + 1 - 0}{h} = -3$

$, f'(1^+) = \lim_{h \to 0^+} \frac{(1+h)^3 - 1 - 0}{h} = 3$

$\therefore f'(1)$ is not exist

$\therefore f'(x) = \begin{cases} -3x^2 & , \quad x < 1 \\ 3x^2 & , \quad x > 1 \end{cases}$

$, f''(x) = \begin{cases} -6x & , \quad x < 1 \\ 6x & , \quad x > 1 \end{cases}$

At $x < 1$ $\qquad f''(x) = 0$ $\qquad \therefore -6x = 0$

$\therefore x = 0$, $f''(x)$ changes before and after zero

$, \because$ It has only one tangent because $f'(0)$ exists

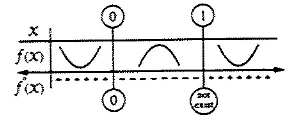
$\therefore$ At $x = 0$ it has inflection point $(0, 1)$

, at $x > 1$ $\qquad f''(x) = 0$ $\qquad \therefore 6x = 0$

$\therefore x = 0$ doesn't satisfy

$f''(x)$ is not exist at $x = 1$

$\therefore$ It has no tangent



The curve is convex upwards at $]0, 1[$

and convex downwards at $]-\infty, 0[$, $]1, \infty[$

The ineflection points at $(0, 1)$

[b] $\int_{-2}^{5} [3f(x) - 6x]\,dx$

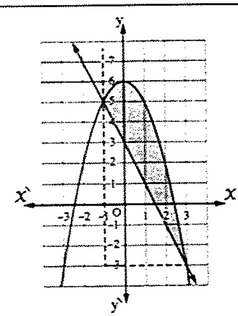
$= 3\int_{-2}^{5} f(x)\,dx - \int_{-2}^{5} 6x\,dx$

$= 3\left[\int_{-2}^{3} f(x)\,dx + \int_{3}^{5} f(x)\,dx\right] - \int_{-2}^{5} 6x\,dx$

$= 3[9 + (-4)] - [3x^2]_{-2}^{5} = 15 - 63 = -48$

4

[a]



$y + x^2 = 6$ $\qquad \therefore y = -x^2 + 6$ $\qquad (1)$

$, y + 2x - 3 = 0$ $\qquad \therefore y = -2x + 3$ $\qquad (2)$

By solving (1) and (2) : $\therefore x = 3$ or $x = -1$

$\therefore$ Area $= \int_{-1}^{3} \left[(-x^2+6) - (-2x+3)\right] dx$

$= \int_{-1}^{3} (-x^2 + 2x + 3)\,dx$

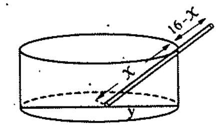
$= \left[-\frac{1}{3}x^3 + x^2 + 3x\right]_{-1}^{3}$

$= \frac{32}{3}$ square unit

[b] $\frac{dx}{dt} = 2$ cm./sec.

$, \because x^2 = y^2 + 81$

(By differentiating with respect to t)



$\therefore 2x\frac{dx}{dt} = 2y\frac{dy}{dt}$ $\qquad \therefore \frac{dy}{dt} = \frac{x}{y}\frac{dx}{dt}$

When the rod reaches the base end.

$y = 12$ cm. , $x = 15$ cm.

$\therefore \frac{dy}{dt} = \frac{15}{12} \times 2 = \frac{5}{2}$ cm./sec.

5

[a] $\frac{d^2y}{dx^2} = 6 - 12x$

$\therefore \frac{dy}{dx} = \int (6 - 12x)\,dx = 6x - 6x^2 + c_1$

$\because$ There is a critical point at $x = 1$

$\therefore 0 = 6 - 6 + c_1$ $\qquad \therefore c_1 = 0$

$\therefore \frac{dy}{dx} = 6x - 6x^2$

Putting : $\frac{dy}{dx} = 0$ $\qquad \therefore x = 0$ or 1

at $x = 1$

$\therefore \left(\frac{d^2y}{dx^2}\right) < 0$ (maximum value)

at $x = 0$

$\therefore \left(\frac{d^2y}{dx^2}\right) > 0$ (minimum value)

$\therefore$ At $x = 0$

There is minimum value $= 4$

$\therefore (0, 4)$ lies on the curve

$, \because y = \int (6x - 6x^2)\,dx = 3x^2 - 2x^3 + c_2$

$\therefore 4 = 3(0) - 2(0) + c_2$ $\qquad \therefore c_2 = 4$

$\therefore y = 3x^2 - 2x^3 + 4$

$\because$ Slope of tangent at $x = -1$

is $\left(\frac{dy}{dx}\right)_{x=-1} = 6(-1) - 6(-1)^2 = -12$

$\therefore$ Slope of normal $= \frac{1}{12}$, at $x = -1$

$\therefore y = 3(-1)^2 - 2(-1)^3 + 4 = 9$

$\therefore$ The point is $(-1, 9)$

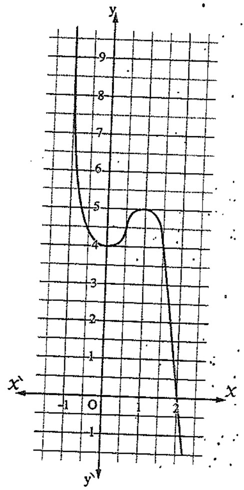
$\therefore$ Equation of normal $\frac{y-9}{x+1} = \frac{1}{12}$

$\therefore x - 12y + 109 = 0$

Putting : $\frac{d^2 y}{d x^2} = 0$ $\quad \therefore x = \frac{1}{2}$ (ineflection)

Find assistance value $f(2) = 0$

x	-1	0	$\frac{1}{2}$	1	2
y	9	4	$\frac{9}{2}$	5	0



[b] $V = \pi \int_0^1 y^2 \, dx = \pi \int_0^1 (x^3 + 1)^2 \, dx$

$= \pi \int_0^1 (x^6 + 2x^3 + 1) \, dx$

$= \pi \left[\frac{1}{7} x^7 + \frac{1}{2} x^4 + x\right]_0^1$

$= \pi \left[\frac{1}{7} + \frac{1}{2} + 1 - (0)\right] = \frac{23}{14} \pi$ cubic unit.